

Please pay close attention to all rounding instructions and as always, NOTA stands for “None of These Answers is correct.” HAVE FUN!

1. Allison loves the study the means and variances of various discrete random variables. What quantity does she have after she computes the sum of the mean and variance of a geometric random variable with probability of success  $p$  and  $0 < p < 1$ ?
  - a.  $\frac{1}{p} - p^2$
  - b.  $\frac{\sqrt{1-p}}{p}$
  - c.  $\frac{1}{p} + p^2$
  - d.  $\frac{1}{p^2}$
  - e. NOTA
  
2. The sum of  $P(\text{Type I error})$  and  $P(\text{Type II error})$  for a two-sample  $t$  test of significance is 0.17. The level of significance for the test was 5% and the  $p$ -value after performing the test was 0.03. What was the power of the test?
  - a. 0.88
  - b. 0.12
  - c. 0.14
  - d. 0.20
  - e. NOTA
  
3. How many ways can 6 keys be arranged on a keychain with a clasp such that the clasp is a fixed point on the keychain such that the keys (once on the keychain) cannot be rotated around the keychain and pass through the clasp?
  - a. 360
  - b. 720
  - c. 2520
  - d. 5040
  - e. NOTA
  
4. In a permutation of the sequence of the letters BALLUPATTHECOURTS, how many distinct permutations contain the sequence BOURTS in that specific order?
  - a. 4,989,600
  - b. 59,875,200
  - c. 58,975,200
  - d. 479,001,600
  - e. NOTA
  
5. How many of the following numerical values are always positive?
  - I. IQR
  - II. Median
  - III. Variance
  - IV. Standard Deviation
  - V. Correlation Coefficient
  - VI. Coefficient of Determination
  - VII. Range
  - VIII. Expected value of a random variable
  - a. 2
  - b. 3
  - c. 4
  - d. 6
  - e. NOTA
  
6. A probability density function is modeled by the graph  $y = \frac{\binom{\lfloor x \rfloor}{3}}{10}$  on the interval  $[3, k]$  where  $\lfloor x \rfloor$  represents the floor function. What is the value of  $k$  that makes this a valid probability density function?
  - a. 5
  - b. 5.5
  - c. 6
  - d. 6.5
  - e. NOTA

7. Which of the following are true statements for independent random variables X and Y each having distinct finite means and distinct finite non-zero standard deviations?
- The variance of their sum is equal to the sum of their variances
  - The variance of their difference is equal to the sum of their variances
  - The mean of their sum is equal to the sum of their means
  - The standard deviation of their sum is equal to the sum of their standard deviations
- a. I, III only    b. II, III, IV only    c. I, II, III only    d. I, III, IV only    e. NOTA
8. Alex's population of interest is all 14 of the MAO statistics competitors at Cypress Bay High School. So, he asks every one of the 14 statistics competitors in MAO how many minutes they spend working on homework in school every day. He finds that the mean of all 14 of these competitors to be 32 minutes and the standard deviation is 3 minutes. Which of the following would be used to estimate the parameter  $\mu$  with 95% confidence where  $\mu$  represents the population mean time spent on homework in school every day by all 14 of these MAO statistics competitors at Cypress Bay High School? Round to three decimal places if necessary.
- a. (30.429, 33.571)    b. (30.329, 33.572)    c. (30.268, 33.732)    d. (30.267, 33.732)    e. NOTA
9. A least squares regression line is used to predict the scores on a math test (represented by Y) given how many pencils the student owns (represented by X). We can safely assume owning pencils is positively correlated with test scores. The regression analysis produces the following sums of squares:  $SST = \sum(y - \bar{y})^2 = 14$ ,  $SSE = \sum(y - \hat{y})^2 = 9.8$ , and  $SSR = \sum(\hat{y} - \bar{y})^2 = 4.2$ . What is the sample correlation coefficient between the test scores and the number of pencils the student owns computed from the statistics above? Round the final answer to three decimal places.
- a. 0.429    b. 0.548    c. 0.655    d. 0.837    e. NOTA
10. There are 2 white balls, 3 black balls, and 4 red balls in a bag. If Tomer is taking out two random balls without replacement from the bag, what is the probability that Tomer takes out two balls of the same color?
- a.  $\frac{5}{18}$     b.  $\frac{29}{81}$     c.  $\frac{13}{18}$     d.  $\frac{7}{24}$     e. NOTA
11. Elijah is playing Dungeons and Dragons with his best friend Eric. He is rolling a 20-sided die because he is a big nerd. He is rolling the die until he gets a prime number. Find the standard deviation of the number of times he needs to roll until he gets his first prime.
- a.  $\frac{2\sqrt{55}}{81}$     b.  $\frac{\sqrt{15}}{2}$     c.  $\frac{\sqrt{15}}{4}$     d.  $\frac{2\sqrt{55}}{9}$     e. NOTA
12. Richard is obsessed with constructing 90 percent confidence intervals to estimate the mean of calculus individual scores since the beginning of time for FAMAT. He will select 69,420 random samples of the same size from the population to construct his intervals. If his method is successful in creating a valid 90% interval, which of the following is captured by approximately 62,478 of the confidence intervals constructed? You may proceed as if all inference conditions are met.
- The Population Median
  - The Population Mean
  - The Sample Mean
  - The Population Standard Deviation
  - NOTA

13. There are 200 kids in SNHS. Of them, there are 60 that are taking AP Physics 1 only, 80 that are taking AP Biology only, and 14 that are taking AP Chemistry only. There are 24 kids that are in AP Biology and AP Chemistry, 15 kids who are in AP Chemistry and AP Physics 1, and 19 who are in AP Physics 1 and AP Biology. The number of kids taking only 2 of any of the classes is 40. All the kids are taking at least one of the 3 classes. Find the number of kids that are in all 3 classes.

- a. 9                      b. 6                      c. 12                      d. 13                      e. NOTA

14. One trial of a simulation has a fair coin tossed five times and we define a success as having at least three heads from the five coin tosses. This simulation is run with 40 trials. What is the probability of having at least 25 successes in the 40 trials of the simulation? Round your final answer to four decimal places.

- a. 0.0769              b. 0.9972              c. 0.0403              d. 0.0366              e. NOTA

**The following is to be used for problems 15 and 16:**

Consider the following probability distributions of independent random variables A and B along with the joint probabilities  $P(A = 1, B = 2) = 0.08$ ,  $P(A = 5, B = 2) = 0.06$ , and  $P(A = 1, B = 4) = 0.03$ .

A	1	3	5	7	9
P(A)	a	0.10	b	0.25	0.30

B	2	4	6	8	10
P(B)	c	d	0.32	0.03	e

15. Find the mean and variance of random variable  $(2A + 3B)$ . The answers below are in the form (mean, variance), and the variance is rounded to 4 decimal places.

- a. (25.08, 95.4576)    b. (25.08, 13.7041)    c. (26.22, 106.7756)    d. (26.22, 187.8026)    e. NOTA

16. Imagine that random variables A and B were in fact not independent and instead have correlation between them of -0.92. Find the standard deviation of random variable  $(A + B)$  rounded to four decimal places.

- a. 3.9442                      b. 5.5630                      c. 1.1819                      d. 5.4512                      e. NOTA

17. Mr. Staubach is concerned about the amount of Diet Coke actually present in his cans of Diet Coke. He randomly selects 50 cans of Diet Coke, and determines that the mean of the sample is 12 fl. ounces per can with a standard deviation of 0.69 oz. He then constructs a 95% confidence interval for  $\mu$ , the true mean amount of fluid ounces of Diet Coke in a can of Diet Coke. Which of the following represents this interval? Round each endpoint of the interval to three decimal places and you may assume all inference assumptions and conditions are met.

- a. (11.804, 12.196)                      c. (11.839, 12.161)                      e. NOTA  
 b. (11.809, 12.191)                      d. (11.836, 12.164)

18. Mr. Novick asks his AP Calculus BC students if they would like to eat Pizza Rolls during the next after school study session. He randomly selects 40 boys and 40 girls from his classes. 21 of the boys and 29 of the girls are in favor of eating Pizza Rolls. Mr. Novick wants to run a significance test at the 1% level to see if there is a statistically significant difference between the proportion of “yes” responses to eating Pizza Rolls of the boys versus the girls. Find the standard error of the sampling distribution for the difference between these two sample proportions needed to run the significance test. Give your final answer as an exact value. You may assume all conditions for the test are met.

- a.  $\frac{\sqrt{3}}{4}$       b.  $\frac{\sqrt{3}}{8}$       c.  $\frac{\sqrt{3}}{16}$       d.  $\frac{\sqrt{3}}{64}$       e. NOTA

19. Events A and B are independent and have the following values: P(A) is equal to the multiplicative inverse of the degrees of freedom present in a Linear Regression significance test with 30 data points, while P(B) is equal to the probability of observing a prime number on a single roll of an ordinary fair die. Find  $|P(A \cup B) - P(A^C | B^C)|$  where  $| |$  is the absolute value of the difference between the probabilities.

- a.  $\frac{25}{56}$       b.  $\frac{31}{56}$       c.  $\frac{13}{29}$       d.  $\frac{18}{29}$       e. NOTA

**The following is a bivariate data sample obtained from a large population. Use this information for question 20**

<b>X</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>Y</b>	<b>4</b>	<b>9</b>	<b>13</b>	<b>20</b>	<b>24</b>	<b>31</b>	<b>30</b>	<b>35</b>	<b>37</b>

20. Elizabeth, after verifying that all necessary conditions for inference have been met, decides to run a test of significance for the following hypotheses:

$H_0: \beta = 3.5$  and  $H_a: \beta > 3.5$  (where  $\beta$  is the slope of the true regression line).

Compute value of the appropriate test statistic rounded to four decimal places (DO NOT round any intermediate steps) then using this test statistic, compute the p-value of the test to three decimal places.

- a. 0.000      b. 0.015      c. 0.017      d. 0.018      e. NOTA

21. A box contains 6 pears and 4 mangos. The pieces of fruit are taken out of the box one at a time and in a random order. What is the probability that the bowl will be empty after the last pear is taken from the box? Round the final answer to four decimal places.

- a. 0.0048      b. 0.0139      c. 0.6000      d. 0.6667      e. NOTA

22. A two-tailed hypothesis test for a population mean has a level of significance of 0.05. The p-value for the test is computed as 0.178. Which of the following statements is/are true?

- I. Using the same sample, a one-tailed test would be statistically significant.
- II. The null hypothesis for this test is not rejected.
- III. The null hypothesis for this test should be rejected.
- IV. The power of this test is 0.95.
- V. The probability of correctly rejecting the null hypothesis for this test is 0.05.

- a. I, II, IV      b. II, IV, V      c. I, III      d. III, V      e. NOTA

23. At a school with 2020 students, it is known that 1250 students take Calculus, 930 take Statistics, and  $x$  take both Calculus and Statistics. What is the difference between the maximum and minimum possible values of  $x$ ?

- a. 320            b. 770            c. 930            d. 1090            e. NOTA

24. The following table displays the Cypress Bay MAO team's performance during one year:

Division	1 <sup>st</sup> – 5 <sup>th</sup> Place Trophies	6 <sup>th</sup> – 10 <sup>th</sup> Place Trophies	11 <sup>th</sup> – 15 <sup>th</sup> Place Trophies
Theta	5	11	2
Alpha	3	2	0
Mu	11	4	1

If you were to do a chi-square test for independence, which of the following would give you the correct (2 decimal places) expected cell count for the number of 6<sup>th</sup> – 10<sup>th</sup> place trophies in the alpha division?

- a. 2.18            b. 2.46            c. 4.33            d. 7.85            e. NOTA

25. Researchers believe that children between the ages of 6 and 10 are watching too much television. They claim that children should watch, on average, no more than 2 hours of television each week. A simple random sample of 14 children is taken from a large local elementary school, and these students were asked how many hours of television they watched during the previous week. The mean of the 14 responses was 2.65 hours with a standard deviation of 0.75 hours. You may assume all necessary conditions for inference are satisfied. Which of the following is the appropriate right-tail p-value for this hypothesis test rounded to four decimal places?

- a. 0.0032            b. 0.0064            c. 0.0006            d. 0.0012            e. NOTA

26. A simple random sample is taken and the resulting sample mean is 22. A 99 percent confidence interval computed for the population mean is  $22 \pm 5$ . Which of the following statements must be true? Assume all inference conditions were satisfied.

- a. 99% of the population measurements are between 17 and 27 inclusive.  
 b. If the population mean is in fact 29, a sample mean of 22 is unlikely to occur.  
 c. If 100 samples are taken, 99 of the sample means would be between 17 and 27 inclusive.  
 d. The probability that the population mean falls between 17 and 27 inclusive is .99.  
 e. NOTA

27. In a test of the hypothesis  $H_0: \mu = 20$  versus  $H_a: \mu > 20$ , the power of this test will be greatest for which of the following combinations of possible alternative, sample size, and significance level?

- a.  $\mu = 21, n = 25, \alpha = .01$   
 b.  $\mu = 22, n = 20, \alpha = .05$   
 c.  $\mu = 21, n = 25, \alpha = .05$   
 d.  $\mu = 22, n = 25, \alpha = .05$   
 e. Impossible to determine from the given information

**Use the information below to answer questions 28 – 30:**

The regression output below references the strong, positive linear relationship between the Weight (lbs.) and Height (inches) of 25-year-old males incarcerated in San Quentin prison. There were 32 prisoners used in the summary information below.

Predictor	Coef.	SE Coef.	T	P
Constant	- 266.53	51.03	- 5.22	0.001
Height	6.1376	0.7353	8.35	0.000
S = 8.641		R-Sq. = 89.7%	R-Sq. (adj.) = 88.4%	

28. What is residual value for a 6' 3'' 25-year-old male in San Quentin who weighs 285 pounds? Round your final answer to two decimal places.

- a. 91.21 lbs.      b. 164.86 lbs.      c. 91.50 lbs.      d. -91.25 lbs.      e. NOTA

29. What is the sample standard deviation of the heights of the prisoners in this study? Do NOT round any intermediate steps and round final answer to three decimal places.

- a. 2.0774 in.      b. 2.111 in.      c. 1.552 in.      d. 1.528 in.      e. NOTA

30. Assuming that all necessary conditions for inference are met, which expression below represents a 99% confidence interval for the slope  $\beta$  of the population regression line? Round the critical value to two decimal places, but do not further round any values you use from the table.

- a.  $6.1376 \pm 2.022075$   
b.  $6.1376 \pm 2.014722$   
c.  $6.1376 \pm 1.808838$   
d.  $6.1376 \pm 2.125686$   
e. NOTA