1. $\boxed{D}$ Take the natural logarithm of the limit and then apply L'Hospital's Rule.

$$\begin{aligned} y &= \lim_{x\to\infty}(1+\frac{\sqrt{2}}{x})^{s\sqrt{2}} \\ \ln y &= \lim_{x\to\infty} x\sqrt{2}\ln(1+\frac{\sqrt{2}}{x}) \\ \ln y &= \sqrt{2}\lim_{x\to\infty}\frac{\ln(x+\sqrt{2})-\ln x}{1/x} \\ \ln y &= \sqrt{2}\lim_{x\to\infty}\frac{\frac{1}{x+\sqrt{2}}-\frac{1}{x}}{-1/x^2} \\ \ln y &= \sqrt{2}\lim_{x\to\infty}\frac{x^2\sqrt{2}}{x(x+\sqrt{2})} \\ \ln y &= \sqrt{2}\sqrt{2}=2 \\ y &= e^2 \end{aligned}$$

2. $\boxed{B}$ Volume of juice is defined by $V = Ah$, where A is the cross sectional area of the cylinder, and h is the height of the juice. Thus, $V' = Ah'$, and so $h' = \dfrac{V'}{A} = \dfrac{48}{\pi\cdot 4^2} = \dfrac{3}{\pi}$

3. $\boxed{A}$ Using the chain rule, the derivative of $e^{\sin x}$ is $(\cos x)e^{\sin x}$, and so the derivative at the given point is $(\cos(\pi/6))e^{\sin \pi/6} = \frac{\sqrt{3e}}{2}$

4. $\boxed{D}$ The slope of the function at $x = 1$ is $2e^3$, and the value of the function at $x = 1$ is $e^3 + 3$. Thus the equation of the tangent line is

$$\begin{aligned} y &= 2e^3(x-1)+e^3+3 \\ y &= 2e^3x - e^3 + 3 \\ y &= (2x-1)e^3+3 \end{aligned}$$

5. $\boxed{A}$ Since $f(x)$ is odd, we know that $\int_{5}^{-5} f(x)dx = 0$ and $\int_{-2}^{2} f(x)dx = 0$. Also, we know that by switching the bounds of the second equation, we get $\int_{-5}^{3} f(x)dx = -14$. Adding the expressions gives

$$\int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx + \int_{5}^{-5} f(x)dx + \int_{-5}^{3} f(x)dx = \int_{-2}^{3} f(x)dx = 10 - 14 = -4$$

6. $\boxed{C}$ First, we plug $x = 1$ into the equation to get that $y = 1$ at that point. Implicitly differentiating the equation gives

$$3x^2 + e^y + xy'e^y + 2yy' = 0$$

Substituting the values for x and y, and then solving for y' gives

$$3 + e + y'e + 2y' = 0$$

$$y'(e+2) = -3 - e$$
$$y' = -\frac{e+3}{e+2}$$

To get the slope of the perpendicular line, we find the negative reciprocal:

$$-\frac{1}{y'} = \frac{e+2}{e+3}$$

7. $\boxed{D}$ To simplify the task, we express the integral as follows

$$\int_{-2}^{3}(x^4+4x^3+6x^2+4x)dx = \int_{-2}^{3}((x+1)^4-1)dx = \frac{(x+1)^5}{5} - x\Big|_{-2}^{3} = \frac{1024-(-1)}{5} - (2-(-3)) = 205-5 = 200$$

.

8. $\boxed{E}$ Simply plugging in $x = 0$ yields that

$$\lim_{x\to 0}\frac{\sin x + \cos x}{\tan^{-1} x} = \frac{\sin 0 + \cos 0}{\tan^{-1} 0} = \frac{1}{0} = \text{DNE}$$

Note that this limit is not ∞, because approaching from the left we get that the limits goes to $-\infty$.

9. $\boxed{C}$ The SA of a hemisphere is equal to half the surface area of a sphere plus the area of it's base, so

$$V_{\text{hemi}}(r) = \frac{1}{2}(4\pi)r^2 + \pi r^2 = 3\pi r^2$$

.

$$V'_{\text{hemi}}(r) = 6\pi r$$

Using differentials, we see that

$$V(r) \approx V(3) + V'(3)(r-3)$$
$$V(4) \approx V(3) + V'(3)(4-3)$$
$$V(4) \approx 27\pi + 18\pi = 45\pi$$

10. $\boxed{D}$ Cost per one piece of candy is given by

$$C(x) = \frac{1}{x}(x^2 + 20x + 288) = x + 20 + \frac{288}{x}$$

Taking the derivative and setting equal to 0, we get

$$C'(x) = 1 - \frac{288}{x^2} = 0$$
$$x = \sqrt{288} = 12\sqrt{2}$$

So the function $C(x)$ attains a minimum at $x = 12\sqrt{2}$. However, Connor can only produce a whole number of candies. If we note that $12\sqrt{2}$ is very close to 17, we will see that $C(x)$ in fact obtains the absolute minimum at $x = 17$ (You can draw the graph of the function and convince yourself that this statement is true).

11. $\boxed{B}$ Our goal is to find the shortest distance between the graphs of $y = \frac{4}{3}x + 10$ and $y = -\frac{1}{5}x^2 + 4x$. There are many ways to do this, but one of the easiest ways is to first note that at the point where the rock is closest to the ceiling, the slope of the graph of it's path is the same as the slope of the line. Thus, we find the derivative of the path and set it equal to the slope of the line:

$$-\frac{2}{5}x + 4 = \frac{4}{3}$$

$$x = \frac{20}{3}$$

The height of the rock at that point is given by

$$y = -\frac{1}{5}\left(\frac{20}{3}\right)^2 + 4\left(\frac{20}{3}\right) = \frac{160}{9}$$

We now find the shortest distance between the ceiling and the point $\left(\frac{20}{3}, \frac{160}{9}\right)$. We do so by using the point-to-line distance formula:

$$d_{min} = \frac{|(\frac{4}{3})(\frac{20}{3}) - \frac{160}{9} + 10|}{\sqrt{(\frac{4}{3})^2 + 1}} = \frac{\frac{10}{9}}{\frac{5}{3}} = \frac{2}{3}$$

12. $\boxed{B}$ Multiplying both sides by the denominator of the fraction gives

$$r(4\sin\theta + 3\cos\theta) = 12$$

$$4r\sin\theta + 3r\cos\theta = 12$$

If we convert this equation to rectangular coordinates, it simply becomes

$$4y + 3x = 12$$

which is just a line. The points corresponding to $\theta = 0$ and $\theta = \frac{pi}{2}$ are $(4, 0)$ and $(0, 3)$. Using Pythagorean theorem, the length of the line is simply 5.

13. $\boxed{D}$

$$f(x) \approx f'(1)(x-1) + f(1)$$

$$f(2) \approx f'(1)(2-1) + f(1) = 1 + 4 = 5$$

14. $\boxed{C}$

$$T = \frac{1}{2}(0-(-1))(5+0) + \frac{1}{2}(1-0)(0+4) + \frac{1}{2}(4-1)(4+(-3)) + \frac{1}{2}(6-4)(-3+(-2)) + \frac{1}{2}(9-6)(-2+2) = 1$$

15. $\boxed{C}$ I. This statement is true. We can extract information using Intermediate Value Theorem. One root is at $x = 0$, there's at least one more in the range $(0, 1)$, at least one more in $(1, 4)$, and at least one more in $(6, 9)$.
II. This is not true. Even though the derivative is 0 at the point, it could also be a relative maximum.
III. This statement is true. Since the function is differentiable, the derivative of the function is continuous. Since the derivative is negative at $x = 0$ and positive at $x = 1$, we can use Intermediate Value Theorem to see that somewhere it is equal to 0.
IV. This statement is true. We can simply use Mean Value Theorem to see that this is true for the specified range.

16. $\boxed{D}$ The area of a hexagon with side length s is given by $\frac{3s^2\sqrt{3}}{2}$. Solving the given equation of the ellipse for y gives that $y = \pm\sqrt{9 - \frac{x^2}{4}}$, and so the side length of the hexagon is given by $s = 2\sqrt{9 - \frac{x^2}{4}}$. The volume of Iris's shape is given by

$$\int_{-6}^{6} \frac{3s^2\sqrt{3}}{2} dx = \int_{-6}^{6} \frac{3\sqrt{3}}{2}(2\sqrt{9 - \frac{x^2}{4}})^2 dx = 432\sqrt{3}$$

17. $\boxed{A}$ Alex's goal is to run to a point where Shreyas's line of sight is blocked by the parabolic wall. If we find a line that's tangent to the wall and goes through Shreyas's location, Alex can run past this line to be safe from Shreyas. There's actually two such lines, but there's only one that's going to be more efficient for Alex. This line is given by the equation $y = 2x - 1$, and is tangent to the wall at $(1, 1)$. The shortest distance that Alex needs to run to get past this line is given by the point-to-line distance formula:

$$d_{min} = \frac{|2(1) - 1(-4) - 1|}{\sqrt{1^2 + 2^2}} = \sqrt{5}$$

18. $\boxed{A}$ One can observe that the sequence $\tan^{-1}(1/n)$ is decreasing and goes to 0 as $n \to \infty$, so by the alternating series test, the original series converges. For absolute convergence, we can use the limit comparison test with the harmonic series $\sum_{n=1}^{\infty}$:

$$\lim_{n\to\infty} \frac{\tan^{-1}(1/n)}{1/n} = 1 \quad \longrightarrow \quad \sum_{n=1}^{\infty} \tan^{-1}(1/n) \quad \text{diverges}$$

Thus the original series is conditionally convergent.

19. $\boxed{D}$ The region is symmetric about the x-axis, so the x coordinate of the centroid is simply 0. The y coordinate of the centroid is given by

$$\overline{y} = \frac{\frac{1}{2}\int_{-1}^{1} 1^2 - (x^2)^2 dx}{\int_{-1}^{1} 1 - x^2 dx} = \frac{4/5}{4/3} = \frac{3}{5}$$

20. $\boxed{A}$ Polar slope is given by

$$\frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{\tan\theta + r/r'}{1 - (r/r')(\tan\theta)} = \frac{1/3 + 1/2}{1 - (1/2)(1/3)} = 1$$

21. $\boxed{C}$ This limit is much easier to do if we instead consider the reciprocal of the fraction. We then have:

$$\lim_{x\to 0} \frac{\sin^2 x - \sin^2 x \cos x + x^4}{x^2 \sin^2 x} = \lim_{x\to 0} \frac{1 - \cos x}{x^2} + \lim_{x\to 0} \frac{x^2}{\sin^2 x} = \frac{1}{2} + 1 = \frac{3}{2}$$

and so the original limit is

$$\lim_{x\to 0} \frac{x^2 \sin^2 x}{\sin^2 x - \sin^2 x \cos x + x^4} = \frac{2}{3}$$

22. $\boxed{C}$ First, we can notice the pattern that

$$A^n = \begin{bmatrix} 2^{n-1} & 0 & 2^{n-1} \\ 0 & 1 & 0 \\ 2^{n-1} & 0 & 2^{n-1} \end{bmatrix}$$

for $n > 0$, also noting that $A^0 = I$. According to the given definition, we have that

$$f(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \sum_{n=1}^{\infty} \frac{1}{n!} \begin{bmatrix} 2^{n-1} & 0 & 2^{n-1} \\ 0 & 1 & 0 \\ 2^{n-1} & 0 & 2^{n-1} \end{bmatrix}$$

Summing each element individually gives us that

$$f(A) = \begin{bmatrix} (e^2+1)/2 & 0 & (e^2-1)/2 \\ 0 & e & 0 \\ (e^2+1)/2 & 0 & (e^2-1)/2 \end{bmatrix}$$

so the sum of all the entries is $2e^2 + e$

23. $\boxed{B}$ Using our definition, we get that

$$f(I_n) = \sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!} I_n = I_n \sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!} = I_n \cdot f(1)$$

where in the last step we used the fact that the sum is equal to the Taylor series for $f(x)$ evaluated at $x = 1$. Then we have that $|f(I_n)| = |I_n \cdot f(1)| = (f(1))^n \cdot |I_n| = (f(1))^n$

24. $\boxed{A}$ Using the substitution $u = x^x$ with $du = x^x(1 + \ln x)dx = u(1 + \ln x)dx$, we get that

$$\int_1^{\infty} \frac{1 + \ln x}{x^x} dx = \int_1^{\infty} \frac{1}{u^2} du = -\frac{1}{u}\Big|_1^{\infty} = 1$$

25. $\boxed{B}$ I. Can be expressed. Using integration by parts,

$$\int xf'(x)dx = xf(x) - \int f(x)dx$$

We know that the antiderivative of $f(x)$ is elementary, and so the original function is also elementary (derivatives are always elementary if the original function is). We therefore see that the whole expression is also elementary.
II. Cannot be expressed. Take $f(x) = xe^{x^2}$ as a counterexample.

$$\int xe^{x^2} dx = \frac{1}{2}e^{x^2} + C$$

while

$$\int x^2 e^{x^2} dx = \frac{1}{2}xe^{x^2} - \frac{1}{2}\int e^{x^2} dx$$

and we know that the last integral cannot be expressed in terms of elementary functions.
III. Can be expressed. Take the substitution $x = f(u)$ with $dx = f'(u)du$, and so

$$\int f^{-1}(x)dx = \int uf'(u)du$$

we already know that this integral is elementary from I.
IV. Cannot be expressed. Take $f(x) = xe^{x^2}$ as a counterexample. The integrals are pretty much the same as the ones in part II.

26. $\boxed{D}$ The volume is given by the integral

$$\pi \int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2} dx$$

If we do integration by parts with $u = \sin^2 x$ and $dv = \frac{1}{x^2}dx$, we get that

$$\pi \int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2} dx = -\pi \frac{\sin^2 x}{x}\bigg|_{-\infty}^{\infty} + \pi \int_{-\infty}^{\infty} \frac{\sin 2x}{x} dx = \pi \int_{-\infty}^{\infty} \frac{\sin 2x}{x} dx$$

Then using the substitution $u = 2x$, we get that

$$\pi \int_{-\infty}^{\infty} \frac{\sin 2x}{x} dx = \pi \int_{-\infty}^{\infty} \frac{\sin u}{u} du$$

The integral from $-\infty$ to ∞ of $\dfrac{\sin u}{u}$ is well-known to have value π. Therefore, the final integral has value π^2.

27. $\boxed{B}$ Rearranging the inequality gives

$$a^x \geq x^a \qquad \Longrightarrow \qquad a^{1/a} \geq x^{1/x}$$

Now, let us consider the function $f(x) = x^{1/x}$. If we find the global maximum of this function, which occurs at some $x = a$, we will know that $f(a) \geq f(x)$ for all positive real x. As $x \to 0$, $f(x) \to 0$, and as $x \to \infty$, $f(x) \to 1$. The derivative of $f(x)$ has a zero at $x = e$, which gives $f(x) = e^{1/e} \approx 1.445$, and so the global maximum of the function occurs at $x = e$. We then know that $a = e$, and so $\lfloor a^2 \rfloor = \lfloor 7.389... \rfloor = 7$

28. $\boxed{A}$ Rewriting the limit gives

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n-1} \frac{1}{\sqrt{\ln(n) - \ln(i)}} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n-1} \frac{1}{\sqrt{-\ln(i/n)}}$$

which is a Riemann sum that can be converted to the integral

$$\int_0^1 \frac{dx}{\sqrt{-\ln x}}$$

Using the substitution $u = \sqrt{-\ln x}$ with $x = e^{-u^2}$ and $dx = -2ue^{-u^2}$ gives

$$\int_0^1 \frac{dx}{\sqrt{-\ln x}} = \int_0^{\infty} 2e^{-u^2} du$$

The last integral is well-known to have value $\sqrt{\pi}$.

29. $\boxed{D}$ Using the substitution $x = \tan\theta$ with $dx = \sec^2\theta d\theta$ gives

$$\int_0^1 \frac{\ln(1+x)}{1+x^2}dx = \int_0^{\pi/4} \ln(1+\tan\theta)d\theta$$

If we let the above integral equal to I, and use the substitution $\theta \to \pi/4 - \theta$, we will get that

$$I = \int_0^{\pi/4} \ln(1+\tan(\pi/4-\theta))d\theta = \int_0^{\pi/4} \ln(1+\frac{1-\tan\theta}{1+\tan\theta})d\theta = \int_0^{\pi/4} \ln(\frac{2}{1+\tan\theta})d\theta$$

Adding the original and the new integrals we get

$$2I = \int_0^{\pi/4} \ln(1+\tan\theta)d\theta + \int_0^{\pi/4} \ln(\frac{2}{1+\tan\theta})d\theta = \int_0^{\pi/4} \ln(2)d\theta = \frac{\pi \ln 2}{4}$$

and therefore $I = \dfrac{\pi \ln 2}{8}$

30. $\boxed{B}$ Dividing the first two equations gives

$$\frac{f'(x)}{g'(x)} = \frac{g(x)\cdot h(x)}{f(x)\cdot h(x)} = \frac{g(x)}{f(x)} \qquad \longrightarrow \qquad f'(x)f(x) = g'(x)g(x)$$

Integrating both sides of the last equation with respect to x gives

$$\int f'(x)f(x)dx = \int g'(x)g(x)dx \qquad \longrightarrow \qquad (f(x))^2 = (g(x))^2 + C$$

where we can use the gives values for f and g to find that $C = 1$. Note that we can repeat the steps above for the last two equations to get that

$$(f(x))^2 = (g(x))^2 + 1 = (h(x))^2$$

Solving for f and h in terms of g gives

$$f(x) = \sqrt{(g(x))^2+1}$$
$$h(x) = -\sqrt{(g(x))^2+1}$$

This is because we know f is positive at $x = 0$, so in order for it to be continuous, f is positive on its entire domain. Similarly, since h is negative at a single point, it must be negative everywhere. Substituting these expressions into the second equation gives

$$g' = -(\sqrt{g^2+1})(\sqrt{g^2+1}) = -g^2 - 1 \qquad \longrightarrow \qquad \frac{g'}{g^2+1} = -1$$

Integrating both sides with respect to x gives

$$\arctan g(x) = -x + C$$

Solving for the constant gives $C = 0$, and therefore $g(x) = -\tan x$. To find the desired quantity, we note that $f(x) = -h(x)$, and so we need do just find $g(\pi/4) = -\tan(\pi/4) = -1$