For all of the following, E. NOTA means "None of the Above". Have fun!

1. Evaluate $\lim_{x\to\infty} (1+\frac{\sqrt{2}}{x})^{x\sqrt{2}}$.

A. 1
B. e
C. $x^{\sqrt{2}}$
D. e^2
E. NOTA

2. Saathvik is pouring orange juice into a cylinder with a radius of 4 inches at a rate of 48 in^3/min (the cylinder has an open top, and is standing on one of its bases). How fast is the level of orange juice rising?

A. 3 in/min
B. 3/π in/min
C. $2\sqrt{3}$ in/min
D. $2\sqrt{3}/\pi$ in/min
E. NOTA

3. Arnav has trouble evaluating the derivative of $e^{\sin(x)}$ at x = π/6. What value should he get if he evaluates it correctly?

A. $\sqrt{3e}/2$
B. $-\sqrt{3e}/2$
C. $-e^{\sqrt{3}/2}/2$
D. $e^{\sqrt{3}/2}/2$
E. NOTA

4. Find the equation of the line tangent to $y = e^{2x+1} + 3$ at x = 1.

A. $y = xe^3$
B. $y = (x-1)e^3 + 3$
C. $y = 2xe^3 + 3$
D. $y = (2x-1)e^3 + 3$
E. NOTA

5. If f(x) is an odd function and $\int_2^5 f(x)\,dx = 10$ and $\int_3^{-5} f(x)\,dx = 14$, what is the value of $\int_{-2}^{3} f(x)\,dx$?

A. -4
B. 0
C. 2
D. 5
E. NOTA

6. What is the slope of the line perpendicular to the tangent line to the graph $x^3 + xe^y + y^2 = e + 2$ at x = 1 in quadrant I?

A. $(e+3)/(e+2)$
B. $-(e+3)/(e+2)$
C. $(e+2)/(e+3)$
D. $-(e+2)/(e+3)$
E. NOTA

7. $\int_{-2}^{3} (x^4 + 4x^3 + 6x^2 + 4x)\, dx =$
   A. 205
   B. 1026/5
   C. 1056/5
   D. 200
   E. NOTA

8. Find $\lim_{x \to 0} \frac{\sin(x) + \cos(x)}{\arctan(x)}$
   A. 1
   B. -1
   C. 2
   D. ∞
   E. NOTA

9. Consider a hemisphere of radius r with a closed top. Using differentials, approximate the surface area of this shape with r = 4 given that the surface area for r = 3 is 27π.
   A. 33π
   B. 36π
   C. 45π
   D. 48π
   E. NOTA

10. Connor is manufacturing candy. He knows that it costs $x^2 + 20x + 288$ dollars to produce x pieces of candy. How many pieces of candy should Connor manufacture so that the cost per one piece of candy is minimized? (Keep in mind Connor can only produce a whole number of candies).
   A. 12
   B. 16
   C. 17
   D. 18
   E. NOTA

11. Jae is standing at the origin of the Cartesian plane, with the x-axis defining the ground. He is very mad at the ceiling, which is defined by the infinite line $y = 4x/3 + 10$. Because he's so mad, he throws a rock at the ceiling, which follows the path of the curve $y = -x^2/5 + 4x$. How close does Jae's rock get to the ceiling?
   A. 1/3
   B. 2/3
   C. 1
   D. 4/3
   E. NOTA

12. Find the arclength of the polar function $r = 12/(4\sin\theta + 3\cos\theta)$ from 0 to $\pi/2$.

A. 4
B. 5
C. 6
D. 8
E. NOTA

For questions 13 - 15, consider the following table of values for a differentiable function f:

x	-1	0	1	4	6	9
f(x)	5	0	4	-3	-2	2
f'(x)	-3	-4	1	0	-2	1

13. Using a linear approximation at $x = 1$, estimate the value of $f(x)$ at $x = 2$.

A. 2
B. 3
C. 4
D. 5
E. NOTA

14. Using trapezoidal rule and all the values given in the table, approximate $\int f(x)\,dx$ from $x = -1$ to $x = 9$.

A. -7
B. -1
C. 1
D. 5/2
E. NOTA

15. Exactly how many of the following statements are necessarily true?

I. f(x) has at least 4 roots on (-1,9)
II. f(x) has a local minimum at $x = 4$
III. $f'(x) = 0$ somewhere on (0,1)
IV. $f'(x) = 4/3$ somewhere on (6,9)

A. 1
B. 2
C. 3
D. 4
E. NOTA

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16. Iris really likes geometry. Her favorite shape is a regular hexagon. Iris also likes ellipses. That's why she creates a solid with a base defined by the graph of $x^2 + 4y^2 = 36$, and cross-sections that are regular hexagons perpendicular to the x-axis, with one of the sides of the hexagon lying in the xy plane. Find the volume of this solid.

A. $108\sqrt{3}$
B. 216
C. 432
D. $432\sqrt{3}$
E. NOTA

17. Shreyas and Alex are on the Cartesian plane. Shreyas is fixed at the location (2,3) and is holding his super-duper PewPew™ laser gun. He is constantly aiming his super-duper PewPew™ laser gun directly at Alex, who is currently located at (1,-4). Luckily, there is an infinite wall built along the graph of the equation $y = x^2$. What is the shortest distance Alex needs to run in order to be able to hide from Shreyas's line of sight and escape his super-duper PewPew™ laser gun?

A. $\sqrt{5}$
B. $\sqrt{10}$
C. 5
D. $5\sqrt{2}$
E. NOTA

18. The series

$$\sum_{n=1}^{\infty}(-1)^n \arctan\left(\frac{1}{n}\right)$$

A. Converges conditionally
B. Converges absolutely
C. Oscillates
D. Diverges
E. NOTA

19. What is the centroid of the region bounded by the graphs of $y = x^2$ and $y = 1$?

A. $\left(\frac{3}{8}, \frac{3}{4}\right)$
B. $\left(\frac{5}{8}, \frac{3}{5}\right)$
C. $\left(\frac{5}{8}, \frac{3}{4}\right)$
D. $\left(\frac{3}{8}, \frac{3}{5}\right)$
E. NOTA

20. Find the slope of the polar graph $r = e^{2\theta}$ at the point where $\tan\theta = 1/3$ and $0 \le \theta \le \pi$.

A. 1
B. 5/6
C. 7/6
D. $2e^{2\arctan(1/3)}$
E. NOTA

21. Find $\lim_{x\to 0} \frac{x^2 \sin^2(x)}{\sin^2(x) - \sin^2(x)\cos(x) + x^4}$

A. 0
B. ∞
C. 2/3
D. 3/2
E. NOTA

For questions 22 - 23, we define non-standard functions of matrices in terms of their Taylor polynomials (centered at 0). In particular, if M is a square matrix,

$$f(M) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} M^n$$

where it converges, and f is an infinitely differentiable function.

22. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Find the sum of all entries in f(A), if $f(x) = e^x$.

A. 3e
B. 5e
C. $2e^2 + e$
D. $4e^2 + e$
E. NOTA

23. Let I_n be the n x n identity matrix and f(x) a function defined for all real numbers. Then $|f(I_n)|$, in terms of f and n, is which of the following?

A. f(1)
B. $(f(1))^n$
C. nf(1)
D. nf(n)
E. NOTA

24. $\int_1^{\infty} \frac{\ln(x)+1}{x^x} dx =$

A. 1
B. e
C. 1/e
D. e^e
E. NOTA

25. Let f(x) be a differentiable, invertible function such that its antiderivative can be expressed in terms of elementary functions (means that f(x) "can be integrated"). Which of the following can also be always expressed in terms of elementary functions?

I. $\int xf'(x)\,dx$
II. $\int xf(x)\,dx$
III. $\int f^{-1}(x)\,dx$
IV. $\int f^2(x)\,dx$

A. I and II only
B. I and III only
C. II, III and IV only
D. I, III and IV only
E. NOTA

26. Let R be the region between the curve y = sin(x)/x and the x-axis. What is the volume of the solid formed when R is rotated about the x-axis?

A. π/2
B. π
C. $\pi^2/2$
D. π^2
E. NOTA

27. There exists a positive real value of a, such that for all positive real x,

$$a^x - x^a \geq 0.$$

What is the value of $[\![a^2]\!]$? ($[\![x]\!]$ is the greatest integer function.)

A. 6
B. 7
C. 8
D. 9
E. NOTA

28. $\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n-1} \frac{1}{\sqrt{\ln(n)-\ln(i)}} =$

A. $\sqrt{\pi}$
B. $\sqrt{2\pi}$
C. $2\sqrt{\pi}$
D. 1/e
E. NOTA

29. $\int_0^1 \frac{\ln(1+x)}{x^2+1} =$

A. $\pi \ln(\sqrt{2}+1)/2$
B. $\pi \ln(\sqrt{2}+1)/4$
C. $\pi \ln(2)/4$
D. $\pi \ln(2)/8$
E. NOTA

30. Let f(x), g(x) and h(x) be differentiable functions such that f(0) = 1, g(0) = 0, and h(0) = -1. Furthermore,

$$f'(x) = g(x)\cdot h(x)$$
$$g'(x) = f(x)\cdot h(x)$$
$$h'(x) = f(x)\cdot g(x)$$

Find f(π/4) + g(π/4) + h(π/4).

A. $2\sqrt{2}$
B. -1
C. $-1 + 2\sqrt{2}$
D. $-1 - 2\sqrt{2}$
E. NOTA