Answers

1. (C)
2. (A)
3. (B)
4. (D)
5. (C)
6. (B)
7. (C)
8. (A)
9. (D)
10. (C)
11. (A)
12. (A)
13. (D)
14. (A)
15. (D)
16. (A)
17. (B)
18. (B)
19. (D)
20. (E)
21. (A)
22. (A)
23. (D)
24. (C)
25. (D)
26. (B)
27. (D)
28. (C)
29. (C)
30. (B)

Solutions

1. Make the substitution $u = \ln x$, $du = \dfrac{dx}{x}$.

$$\begin{aligned}\int_1^e \frac{(\ln x)^{2021}}{x}\,dx &= \int_0^1 u^{2021}\,du \\ &= \frac{u^{2022}}{2022}\Big|_0^1 \\ &= \frac{1}{2022}\end{aligned}$$

$\boxed{C}$

2. Write $f_n(x) = \dfrac{(1-x^n)^3}{(1-x^3)^n}$ for $x \in [0,1]$. The Bernoulli inequality states that $(1+a)^n \geq 1 + an$ for all $a > -1$ and positive n. Thus, we have

$$f_n(x) \leq \frac{(1-x^n)^3}{1+nx^3} \leq \frac{1}{nx^3}.$$

We then have

$$\int_0^1 \frac{1}{nx^3}\,dx \to \left\{ \begin{array}{l} z = \sqrt[3]{n}x \\ dz = \sqrt[3]{n}\,dx \end{array} \right\} \to \frac{1}{\sqrt[3]{n}} \int_0^{\sqrt[3]{n}} \frac{1}{1+z^3}\,dz.$$

We note that the improper integral $\displaystyle\int_0^\infty \frac{1}{1+z^3}\,dz$ converges by comparison to $\displaystyle\int_0^\infty \frac{1}{1+z}\,dz$. Let this value be A. Thus, we have

$$0 \leq \lim_{n\to\infty} \int_0^1 f_n(x)dx \leq \lim_{n\to\infty} \frac{1}{\sqrt[3]{n}} \int_0^{\sqrt[3]{n}} \frac{1}{1+z^3}\,dz \leq \lim_{n\to\infty} \frac{A}{\sqrt[3]{n}} = 0.$$

$\boxed{A}$

3. Note that the area of a regular octagon with side length a is $a^2(2+2\sqrt{2})$. Now, we must split the given region and evaluate two separate integrals. The first region is bounded above by the line $y = x+7$ and below by the line $y = -x-1$ for $x \in [-4, 0]$. The second region is bounded above by the line $y = x+7$ and below by the line $y = 3x - 1$ for $x \in [0, 4]$. Thus, the volume of Andrew's cake is:

$$\begin{aligned}&(2+2\sqrt{2})\left(\int_{-4}^0 ((x+7)-(-x-1))^2\,dx + \int_0^4 ((x+7)-(3x-1))^2\,dx\right) \\ &= (2+2\sqrt{2})\left(\int_{-4}^0 ((2x+8)^2\,dx + \int_0^4 (2x-8)^2\,dx\right) \\ &= (2+2\sqrt{2})\left(\left(\frac{(2x+8)^3}{6}\right)\Big|_{-4}^0 + \left(\frac{(2x-8)^3}{6}\right)\Big|_0^4\right) \\ &= \frac{1024(1+\sqrt{2})}{3}\end{aligned}$$

$\boxed{B}$

4. Note that $\sin ax = \dfrac{e^{-aix} - e^{aix}}{2i}$ so $\dfrac{\sin 4x}{\sin x} = \dfrac{e^{-4ix} - e^{4ix}}{e^{-ix} - e^{ix}} = e^{3ix} + e^{ix} + e^{-ix} + e^{-3ix}$. The antiderivative of this is $i\left(\dfrac{e^{-3ix} - e^{3ix}}{3} + (e^{-ix} - e^{ix})\right)$, which evaluated at the bounds gives a value of $\dfrac{4}{3} - 0 = \dfrac{4}{3}$. $\boxed{D}$

5. Note that if we look at a circular cross-section of the planet with Jackson at a distance of 4100 miles from the center, we can draw two tangents to the circle. Using right triangles, we have the length of these tangents are each 900 miles. Now, if we plot our diagram on the Cartesian plane, with Jackson on the positive x-axis and the circle with center $(0, 0)$, we can see that the cross section of the area that Jackson can see is the smaller arc subtended by the two points of tangency described before. Using similar triangles, we can see that the line through these two points of tangency is $x = \dfrac{160000}{41}$, and thus $r - x = \dfrac{4000}{41}$. Thus, we simply need to find the surface area of revolution for this circle with the given region.

   Consider the function $f(x) = \sqrt{r^2 - x^2}$, and a point $(x, 0)$ where $x \in [0, r]$. Our desired area is found by finding the surface area of revolution of $f(x)$ on $[x, r]$.

$$\begin{aligned}\int_x^r 2\pi \cdot f(x)\sqrt{1 + (f'(x))^2}\, dx &= \int_x^r 2\pi\sqrt{r^2 - x^2}\sqrt{1 + \frac{x^2}{r^2 - x^2}}\, dx \\ &= \int_x^r 2\pi r\, dx \\ &= 2\pi r(r - x)\end{aligned}$$

   Taking the surface area of a sphere as $4\pi r^2$, we have that the fraction of the area of the planet that Jackson can see is $\dfrac{2\pi r(r - x)}{4\pi r^2} = \dfrac{r - x}{2r}$. We simply plug in $r = 4000, r - x = \dfrac{4000}{41}$ as determined before, and we have that the fraction of the planet Jackson can see is $\dfrac{\frac{4000}{41}}{2 \cdot 4000} = \dfrac{1}{82} \approx 0.012195$. Thus, 1.2% is closest to this. $\boxed{C}$

   For questions 6 and 7, we must first derive the parametric equations for a cycloid with a circle of radius 1. Let θ be the angle through which our circle has rotated through since its initial value of $\theta = 0$. We note that the center of the wheel is at the point $(\theta, 1)$, and thus the point along the circle is displaced by $-\sin\theta$ horizontally and $-\cos\theta$ vertically. Thus, our parametric equations are $x = \theta - \sin\theta, y = 1 - \cos\theta$.

6. To compute the volume of the revolved solid, we use the washer method:

$$\begin{aligned}V = \pi\int_0^\pi y^2\, dx &= \int_0^\pi y(\theta)^2 \cdot x'(\theta)\, d\theta \\ &= \pi\int_0^\pi (1 - \cos\theta)^3\, d\theta \\ &= \pi\int_0^\pi 1 - 3\cos\theta + 3\cos^2\theta - \cos^3\theta\, d\theta \\ &= \pi\int_0^\pi 1 + 3\cos^2\theta\, d\theta \\ &= \pi \cdot \frac{10x + 3\sin 2x}{4}\Big|_0^\pi \\ &= \frac{5\pi^2}{2}\end{aligned}$$

   $\boxed{B}$

7. To compute the surface area of the revolved solid, we use the following:

$$\begin{aligned}
SA &= 2\pi \int_0^{\pi} y(\theta)\sqrt{x'(\theta)^2 + y'(\theta)^2}\, d\theta \\
&= 2\pi \int_0^{\pi} (1-\cos\theta)\sqrt{(1-\cos\theta)^2 + \sin^2\theta}\, d\theta \\
&= 2\pi \int_0^{\pi} (1-\cos\theta)\sqrt{2-2\cos\theta}\, d\theta \\
&= 4\pi \int_0^{\pi} (1-\cos\theta)\sin\frac{\theta}{2}\, d\theta \\
&= 4\pi \int_0^{\pi} \sin\frac{\theta}{2}\, d\theta - 4\pi \int_0^{\pi} \cos\theta \sin\frac{\theta}{2}\, d\theta \\
&= 8\pi - 2\pi \int_0^{\pi} (\sin\frac{3x}{2} - \sin\frac{x}{2})\, d\theta \\
&= 8\pi - 2\pi \cdot \left(-\frac{4}{3}\right) \\
&= \frac{32\pi}{3}
\end{aligned}$$

$\boxed{C}$

8. Let J be a random variable which represents Jack's score at the end of the game, and define D similarly for Danger. Also, define J_i and D_i to be random variables which represent Jack's score and Danger's score in round i, respectively. Note that the expected value of any given roll of Jack's die is 4 and the expected value of any given roll of Danger's die is $\frac{5}{2}$. We need $\mathbb{E}[J] = \mathbb{E}[D]$. If the game has r rounds, then

$$\begin{aligned}
\mathbb{E}[J] &= \mathbb{E}[J_1 + J_2 + \cdots + J_r] = \mathbb{E}[J_1] + \mathbb{E}[J_2] + \cdots + \mathbb{E}[J_r] \\
&= 4 + 4\cdot 4 + \cdots + 4 \cdot 4 = 4 + 16\,(r-1)\,,
\end{aligned}$$

and similarly

$$\begin{aligned}
\mathbb{E}[D] &= \mathbb{E}[D_1 + D_2 + \cdots + D_r] = \mathbb{E}[D_1] + \mathbb{E}[D_2] + \cdots + \mathbb{E}[D_r] \\
&= \frac{5}{2} + \frac{5}{2}\cdot\left(\frac{5}{2}+k\right) + \cdots + \frac{5}{2}\cdot\left(\frac{5}{2}+k\right) = \frac{5}{2} + \frac{5}{2}\left(\frac{5}{2}+k\right)(r-1)\,.
\end{aligned}$$

Therefore, we need

$$4 + 16(r-1) = \frac{5}{2} + \frac{5}{2}\left(\frac{5}{2} + k\right)(r-1) \iff 16 + 64(r-1) = 10 + (25 + 10k)(r-1)$$

$$\iff (10k - 39)(r-1) = 6,$$

which only has one solution in the positive integers, namely $(k, r) = (4, 7)$. Thus, 7 rounds should be played per game. $\boxed{A}$

9. Let $a = BC, b = AC, c = AB$ and $\theta = \angle A$. At time $t = 1$, we have $b = 3, c = 6$, and $\theta = \frac{3\pi}{4}$. Using Law of Cosines, we can compute a to be $3\sqrt{5 + 2\sqrt{2}}$. Also, note that $b' = -1, c' = 3$, and $\theta' = \frac{\pi}{4}$. We can differentiate the equation for Law of Cosines with respect to time to obtain the following:

$$\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos\theta \\
2aa' &= 2bb' + 2cc' - 2(b'c + cb')\cos\theta + 2bc\theta'\sin\theta \\
a' &= \frac{bb' + cc' - (b'c + cb')\cos\theta + bc\theta'\sin\theta}{a} \\
k &= \frac{15 + 12\cdot\frac{\sqrt{2}}{2} + 18\cdot\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2}}{3\sqrt{5 + 2\sqrt{2}}} \\
k &= \sqrt{5 + 2\sqrt{2}} + \frac{3\sqrt{2}}{\sqrt{5 + 2\sqrt{2}}}\cdot\pi
\end{aligned}$$

Thus, we have $k = 5 + 2\sqrt{2} + d\pi$, thus $a = 5$, $b = 2$, and $c = 2$, so $a + b + c = 9$.

$\boxed{D}$

10. We can use the Sophie-Germain factorization and partial fraction decomposition to obtain the following:

$$\begin{aligned}
\int_0^1 \frac{1}{x^4 + 4}\,dx &= \int_0^1 \frac{1}{(x^2 - 2x + 2)(x^2 + 2x + 2)}\,dx \\
&= \frac{1}{8}\int_0^1 \frac{x + 2}{x^2 + 2x + 2} - \frac{x - 2}{x^2 - 2x + 2}\,dx \\
&= \frac{1}{8}\int_0^1 \frac{x + 1}{x^2 + 2x + 2} + \frac{1}{(x + 1)^2 + 1} - \frac{x - 1}{x^2 - 2x + 2} + \frac{1}{(x - 1)^2 + 1}\,dx \\
&= \frac{1}{8}\left(\frac{1}{2}\ln\left(\frac{x^2 + 2x + 2}{x^2 - 2x + 2}\right) + \tan^{-1}(x + 1) + \tan^{-1}(x - 1)\right)\Big|_0^1 \\
&= \frac{1}{16}(\ln 5 + \arctan 2)
\end{aligned}$$

$\boxed{C}$

11. Let L be Sameera's limit. Consider $\ln L = \lim_{x\to 0}\frac{\ln\sin x - \ln x}{1 - \cos x}$. Applying L'Hôpital's Rule, we have $\ln L = \lim_{x\to 0}\frac{\cot x - \frac{1}{x}}{\sin x} = \lim_{x\to 0}\frac{x\cot x - 1}{x\sin x} = \lim_{x\to 0}\frac{x\cos x - \sin x}{x\sin^2 x}$. Applying L'Hôpital's Rule again, we have $\lim_{x\to 0}\frac{-x}{2x\cos x + \sin x} = \lim_{x\to 0}\frac{-1}{2\cos x + \frac{\sin x}{x}} = -\frac{1}{\lim_{x\to 0}(2\cos x) + \lim_{x\to 0}\left(\frac{\sin x}{x}\right)} = -\frac{1}{2 + 1} = -\frac{1}{3}$.

Thus, $L = e^{-1/3}$. $\boxed{A}$

12. The Taylor series of $\sin(x+ax^3)$ begins $(x+Gx^3) - \dfrac{(x+Gx^3)^3}{3!} + \dfrac{(x+Gx^3)^5}{5!} - \dots$. Expanding, this is equal to $x + \left(G - \dfrac{1}{6}\right)x^3 + \left(\dfrac{1}{120} - \dfrac{G}{2}\right)x^5 + \dots$. The limit will only exist when the x and x^3 terms of the numerator cancel out, so $G = \dfrac{1}{6}$ for the minimum degree of a term of the numerator to be equal to 5. Consequently, the coefficient of the x^5 term of the numerator is the value of the limit, which is $J = \dfrac{3}{40}$. $(GJ)^{-1} = 80$. $\boxed{A}$

13. Call the limit L. Then, $L = \lim_{x\to 0}\left(\dfrac{\sin 3x + ax}{x^3} + b\right) = \lim_{x\to 0}\left(\dfrac{3\cos 3x + a}{3x^2} + b\right)$. For this limit to exist, we must have the numerator of our fraction equal 0 at $x = 0$. Thus, $3\cos(3\cdot 0) + a = 0 \implies a = -3$. Now, we proceed by computing $\lim_{x\to 0}\left(\dfrac{3\cos 3x - 3}{3x^2}\right) = \lim_{x\to 0}\dfrac{-9\sin 3x}{6x} = -\dfrac{9}{2}$. Thus, $b = \dfrac{9}{2}$ and $a + b = -3 + \dfrac{9}{2} = \dfrac{3}{2}$. $\boxed{D}$

For questions 14-17, note that $\mathcal{R}$ is bounded below by the x-axis and above by $y = 2x - 3$ for $x \in [\frac{3}{2}, 3]$ and above by $y = -x^2 + 8x - 12$ for $x \in [3, 6]$.

14.

$$\begin{aligned}
A &= \int_{\frac{3}{2}}^{3} (2x-3)\,dx + \int_{3}^{6} (-x^2 + 8x - 12)\,dx \\
&= x^2 - 3x\Big|_{\frac{3}{2}}^{3} + \left(\frac{-x^3}{3} + 4x^2 - 12x\right)\Big|_{3}^{6} \\
&= \frac{9}{4} + 9 \\
&= \frac{45}{4}
\end{aligned}$$

$\boxed{A}$

15. Using the washer method, we have:

$$\begin{aligned}
V &= \pi\int_{\frac{3}{2}}^{3} (2x-3)^2\,dx + \pi\int_{3}^{6} (-x^2 + 8x - 12)^2\,dx \\
&= \pi\int_{\frac{3}{2}}^{3} (4x^2 - 12x + 9)\,dx + \pi\int_{3}^{6} (x^4 - 16x^3 + 88x^2 - 192x + 144)\,dx \\
&= \pi\left(\frac{4x^3}{3} - 6x^2 + 9x\right)\Big|_{\frac{3}{2}}^{3} + \pi\left(\frac{x^5}{5} + 4x^4 + \frac{88x^3}{3} - 96x^2 + 144x\right)\Big|_{3}^{6} \\
&= \pi\cdot\left(\frac{9}{2} + \frac{153}{5}\right) \\
&= \frac{351\pi}{10}
\end{aligned}$$

$\boxed{D}$

16. Using the method of cylindrical shells, we have:

$$\begin{aligned} V &= 2\pi \int_{\frac{3}{2}}^{3} x \cdot (2x-3)\,dx + 2\pi \int_{3}^{6} x \cdot (-x^2+8x-12)\,dx \\ &= 2\pi \int_{\frac{3}{2}}^{3} (2x^2-3x)\,dx + 2\pi \int_{3}^{6} (-x^3+8x^2-12x)\,dx \\ &= 2\pi \left(\frac{2x^3}{3} - \frac{3x^2}{2}\right)\Big|_{\frac{3}{2}}^{3} + 2\pi \left(-\frac{x^4}{4} + \frac{8x^3}{3} - 6x^2\right)\Big|_{3}^{6} \\ &= 2\pi \left(\frac{45}{8} + \frac{153}{4}\right) \\ &= \frac{351\pi}{4} \end{aligned}$$

$\boxed{A}$

17. Note that the total area of $\mathcal{R}$ is $\frac{45}{4}$, and the area contributed from $x \in [\frac{3}{2}, 3]$ is $\frac{9}{2}$, less than half of the total area, $\frac{45}{4}$. Thus, we know $a > 3$. We must have $\int_a^6 (-x^2+8x-12)\,dx = \frac{45}{8}$, so this yields $\frac{a(a-6)^2}{3} - \frac{45}{8} = 0$. Using the intermediate value theorem and plugging in $a = 3$ and $a = 4$, we see that the value of this expression changes sign, so there must exist a root a in the interval $[3, 4]$. $\boxed{B}$

18. Let the initial speed of the projectile be v_0. We have that the vertical acceleration of the projectile is $a_y(t) = -g = -10$. Thus, $v_y(t) = v_0 \sin 60^\circ - 10t$ and $s_y(t) = 100 + \frac{v_0\sqrt{3}}{2}t - 5t^2$. We must have that $s_y(10) = 0$, so $100 + \frac{v_0\sqrt{3}}{2} \cdot 10 - 500 = 0 \implies v_0 = \frac{80\sqrt{3}}{3}$. $\boxed{B}$

19. Since the projectile has no horizontal acceleration, we know that $v_x(t) = v_0 \cos 60^\circ = \frac{40\sqrt{3}}{3}$, so $s_x(t) = \frac{40t\sqrt{3}}{3}$. Thus, our answer is given by $s_x(10) = \frac{400\sqrt{3}}{3}$. $\boxed{D}$

20. We simply need to find the maximum value of the expression $s_y(t)$ as described before. This occurs when $v_y'(t) = 40 - 10t = 0$. This yields $t = 4$, and thus our answer is $s_y(4) = 100 + 40 \cdot 4 - 5 \cdot 4^2 = 180$ meters. $\boxed{E}$

21. We will determine the work Tyger must perform to lift the rope and the monkey when both have been pulled up a distance of x above the ground across an infinitesimally small distance dx. First, note that the graph of the monkey's mass, m_1, is linear with respect to x, passing through the two points $(x, m_1) = (0, 20)$ and $(x, m_1) = (100, 10)$. Thus, the equation for the monkey's mass a distance of x above the ground is $m_1(x) = 20 - \frac{x}{10}$. Now, since the chain has linear density as well, the mass of the chain, m_2, is also linear with respect to x, passing through the points $(x, m_2) = (0, 1000)$ and $(x, m_2) = (100, 0)$. Thus, the equation for the chain's mass a distance of x above the ground is $m_2(x) = 1000 - 10x$.
Since work is equal to force times distance, we can compute the gravitational force of the chain-monkey system as $F_g(x) = mg = (m_1(x) + m_2(x)) \cdot 10 = 10200 - 101x$. Since Tyger must carry this a distance of dx upwards, we have the total work is given by $W = \int_0^{100} F_g(x)\,dx = \int_0^{100} (10200 - 101x)\,dx = \left(10200x - \frac{101x^2}{2}\right)\Big|_0^{100} = 515000$ joules. Thus, our answer, in kilojoules, is 515. $\boxed{A}$

22. We can begin by inscribing the largest possible triangle into a circle of radius 4, an equilateral triangle. The area of this equilateral triangle with circumradius 4 is $12\sqrt{3}$. Since our ellipse can be dilated horizontally by a factor of 2 from our circle of radius 4, the area of our largest such triangle scales accordingly. Thus, our answer is $2 \cdot 12\sqrt{3} = 24\sqrt{3}$. $\boxed{A}$

23. We can write $x(t) = \sin^4(t) = \cos^4(t) = (\sin^2(t) + \cos^2(t))^2 - 2\sin^2(t)\cos^2(t) = 1 - \dfrac{\sin^2(2t)}{2}$. Since the jerk is the third derivative, we compute the following:

$$\begin{aligned} x'(t) &= -2\sin(2t)\cos 2t = -\sin 4t \\ x''(t) &= -4\cos 4t \\ x'''(t) &= 16\sin 4t \end{aligned}$$

Thus, $x'''\left(\dfrac{5\pi}{12}\right) = 16\sin\left(\dfrac{5\pi}{3}\right) = -8\sqrt{3}$. $\boxed{D}$

24. The tangent line to the graph $P(t)$ is given by $y(t) = P'(4)(t-4) + P(4) = 23(t-4) + 44$. The value we are looking for is $y(5) = 23 + 44 = 67$. $\boxed{C}$

25. Jackson's speed at time t is given by:

$$\sqrt{(x'(t))^2 + ((y'(t))^2} = \sqrt{(\cos t^2 - 2t^2\sin t^2)^2 + (2t\cos t - t^2\sin t))^2}$$

We then evaluate this expression at $t = 1$.

$$\begin{aligned} &= \sqrt{(\cos 1 - 2\sin 1)^2 + (2\cos 1 - \sin 1)^2} \\ &= \sqrt{5\cos^2 1 + 5\sin^2 1 - 4 \cdot 2\sin 1\cos 1} \\ &= \sqrt{5 - 4\sin 2} \end{aligned}$$

$\boxed{D}$

26. Since $f(x) = (x+2)^2$ and $f'(x) = 2(x+2)$, in general, the n^{th} iteration of Newton's method is given by $x_n = x_{n-1} - \dfrac{f(x_{n-1})}{f'(x_{n-1})} = x_{n-1} - \dfrac{x_{n-1}+2}{2}$. With this, we have:

$$\begin{aligned} x_1 &= \pi - \frac{\pi+2}{2} = \frac{\pi}{2} - 1 \\ x_2 &= \left(\frac{\pi}{2} - 1\right) - \frac{\frac{\pi}{2} - 1 + 2}{2} = \frac{\pi}{4} - \frac{1}{2} \\ a = x_3 &= \left(\frac{\pi}{4} - \frac{1}{2}\right) - \frac{\frac{\pi}{4} - \frac{1}{2} + 2}{2} = \frac{\pi}{8} - \frac{1}{4} \end{aligned}$$

Thus, $\lfloor 64a \rfloor = \lfloor 8\pi - 16 \rfloor = 9$ $\boxed{B}$

27. We can compute the volume removed by calculating the volume of each of the spherical caps along with the cylindrical shaped created by the rest of the hole. Graphing the largest cross-section of the globe as $f(x) = \sqrt{1-x^2}$, we can compute the volume of revolution of $f(x)$ on $[\frac{1}{2}, 1]$ for each of the spherical caps.

$$\begin{aligned} \pi\int_{\frac{1}{2}}^{1} (1-x^2)\,dx &= \pi(x - \frac{x^3}{3})\Big|_{\frac{1}{2}}^{1} \\ &= \frac{5\pi}{24} \end{aligned}$$

The cylindrical hole bored out has radius $\dfrac{\sqrt{3}}{2}$ and height 1, thus its volume is $\pi \cdot \left(\dfrac{\sqrt{3}}{2}\right)^2 \cdot 1 = \dfrac{3\pi}{4}$.

Thus, our total volume is $2 \cdot \dfrac{5\pi}{24} + \dfrac{3\pi}{4} = \dfrac{7\pi}{6}$. $\boxed{D}$

28. At the n^{th} iteration of the fractal we add $3 \cdot 4^{n-1}$ additional triangles, each with area $\frac{\sqrt{3}}{4}\left(\frac{s_0}{3^n}\right)^2$ for a total added area of $\frac{\sqrt{3}}{4}s_0^2\left(\frac{3\cdot 4^{n-1}}{9^n}\right)$. After n iterations, the total area will be $\frac{\sqrt{3}}{4}s_0^2\left(1+\sum_{k=1}^{n}\frac{3\cdot 4^{k-1}}{9^k}\right)$. As n approaches ∞, the total area is $\frac{\sqrt{3}}{4}s_0^2\left(1+\frac{3/9}{1-4/9}\right) = \frac{2s_0^2\sqrt{3}}{5}$. Setting $s_0 = 3^{1/4}$ gives a total area of $\frac{6}{5}$. $\boxed{C}$

29. Each individual side is broken into $N = 4$ pieces, each $\epsilon = \frac{1}{3}$ as long as the sides in the previous iteration. Thus, the fraction is equal to $\frac{\ln 4}{\ln 3}$. By change of base this is also equal to $\frac{\log_{10} 4}{\log_{10} 3}$. Taking these values to three decimal places each, approximations for the dimension of the Koch snowflake are $\frac{1386}{1099}$ and $\frac{602}{477}$ respectively. $\boxed{C}$

30. As per the hint, we use integration by parts and partial fraction decomposition as follows:

$$\begin{aligned}
\int_{-1}^{0}\frac{\ln(x^2+1)}{(x+2)^2}\,dx &= -\frac{\ln(x^2+1)}{x+2}\bigg|_{-1}^{0} + \int_{-1}^{0}\frac{2x}{(x+2)(x^2+1)}\,dx \\
&= \ln 2 + \frac{2}{5}\int_{-1}^{0}\left(\frac{2x+1}{x^2+1} - \frac{2}{x+2}\right)dx \\
&= \ln 2 + \frac{2}{5}\left(\ln(x^2+1)\bigg|_{-1}^{0} + \tan^{-1}x\bigg|_{-1}^{0} - 2\ln(x+2)\bigg|_{-1}^{0}\right) \\
&= \ln 2 + \frac{2}{5}\left(-\ln 2 + \frac{\pi}{4} - 2\ln 2\right) \\
&= \frac{\pi - 2\ln 2}{10}
\end{aligned}$$

$\boxed{B}$