NOTA denotes "None of the Above". Good luck, and have fun! ~

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1) Let A be the area of an equilateral triangle with side length 2020, and let P be the perimeter of an equilateral triangle with side length 2020. Ignoring units, compute $\frac{A}{P}$.

A) $\dfrac{505\sqrt{3}}{3}$ B) $\dfrac{1010\sqrt{3}}{3}$ C) $505\sqrt{3}$ D) $1010\sqrt{3}$ E) NOTA

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2) Let $A_n(s)$ be the area of a regular n-gon with side length s. Compute $\lim\limits_{n\to\infty} \dfrac{A_n(s)}{n^2}$.

A) $\dfrac{s^2}{2\pi}$ B) $\dfrac{s^2}{\pi}$ C) πs^2 D) $2\pi s^2$ E) NOTA

For Questions 3-4, consider the following piecewise function:

$$g(x) = \begin{cases} x+2 & 0 \le x < 1 \\ 2x+1 & 1 \le x < 2 \\ 3x-1 & 2 \le x \le 3 \end{cases}$$

3) Compute the area of the region R bounded by $g(x)$ and the x-axis.

A) 12 B) $\dfrac{25}{2}$ C) 13 D) $\dfrac{27}{2}$ E) NOTA

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4) Compute the volume of the solid formed when the region R defined in Question 3 is revolved about the x-axis.

A) $\dfrac{191\pi}{3}$ B) $\dfrac{193\pi}{3}$ C) 65π D) $\dfrac{197\pi}{3}$ E) NOTA

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5) A triangle has two side lengths of 5 and 8 units. Compute the maximum possible area of the triangle (in units squared).

A) 20 B) $5\sqrt{39}$ C) 39 D) 40 E) NOTA

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6) Consider a function $g(x)$ that is continuous on the interval $[0, 2021]$ and differentiable on $(0, 2021)$. If $\int_0^{2021} g(x)\, dx = 30$ and $\int_0^{2021} |g(x)|\, dx = 40$, compute the area bounded above by $g(x)$ and below by the x-axis on the interval $[0, 2021]$.

A) 10 B) 35 C) 40 D) 70 E) NOTA

7) Let C be the cone of maximal volume that can be inscribed in a cylinder with height 6 feet and radius 8 feet. Compute the surface area of C (in feet squared).

A) 12π B) 32π C) 44π D) 56π E) NOTA

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8) $e(x)$ is an even function and $o(x)$ is an odd function such that the following are true.

$$\int_0^2 e(x)\,dx = 5 \qquad \int_{-5}^2 o(x)\,dx = -6 \qquad \int_{-2}^{-7} e(x)\,dx = -2 \qquad \int_{-7}^5 o(x)\,dx = 9$$

Assuming that the domains of $e(x)$ and $o(x)$ are both $\mathbb{R}$, compute $\int_{-2}^{7} (2e(x) - 5o(x))\,dx$.

A) 31 B) 39 C) 46 D) 54 E) NOTA

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9) Let R be the region bounded by the graphs of $x = 10 - 2y^2$ and $x = 2y^2 - 6$. Let S be the solid whose cross sections are regular hexagons whose bases are inscribed in R and perpendicular to the x-axis. Compute the volume of S.

A) $128\sqrt{3}$ B) $144\sqrt{3}$ C) $192\sqrt{3}$ D) $216\sqrt{3}$ E) NOTA

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10) Find the volume of the 3-dimensional paraboloid given by the equation $x^2 + y^2 + z = 9$ above the plane $z = 0$.

A) 18π B) $\frac{81\pi}{4}$ C) 36π D) $\frac{81\pi}{2}$ E) NOTA

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11) Connor is playing around with interesting graphs and decides to approximate the area bound by the curve $g(x) = x^3 + 3x^2 + 3x + 5$ and the x-axis on the interval $[0, 2020]$ using the trapezoidal method and 2020 equal subintervals. Assuming Connor got the right answer, compute the remainder when Connor's answer is divided by 1000.

A) 310 B) 510 C) 710 D) 810 E) NOTA

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12) It is a known fact that the area bound by the curve $f(x) = e^{-x^2}$ and the x-axis is $\sqrt{\pi}$. Let Q be the first quadrant region bounded by the curve $g(x) = 2020^{-\frac{x^2}{2} - \frac{1}{2x^2}}$ and the x-axis. Let V be the volume of the solid formed when Q is revolved around the x-axis. If V can be written in the form $\frac{A\pi^B}{C(\ln D)^E}$ where A and C are coprime positive integers, B and E are rational numbers, and D is an integer that is not a perfect power, compute the remainder when $A \cdot B \cdot C \cdot D \cdot E$ is divided by 10000.

A) 2000 B) 4000 C) 6000 D) 8000 E) NOTA

For Questions 13-16, define $g(x) = -x^2 + 6x - 1$. Additionally, let R be the region bounded above by $g(x)$ and below by the x-axis.

13) If the area of R can be written in the form $\frac{m\sqrt{n}}{p}$ where m and p are relatively prime positive integers and n is a squarefree positive integer, find $m + n + p$.

A) 21 B) 37 C) 47 D) 69 E) NOTA

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14) If the volume of the solid formed when R is revolved about the x-axis can be written in the form $\frac{a\pi\sqrt{b}}{c}$ where a and c are relatively prime positive integers and b is a squarefree positive integer, find the remainder when $a + b + c$ is divided by 100.

A) 41 B) 65 C) 69 D) 81 E) NOTA

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15) If the sum of the coordinates of the centroid of R can be written in the form $\frac{u}{v}$ where u and v are relatively prime positive integers, find $u + v$.

A) 28 B) 36 C) 52 D) 84 E) NOTA

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16) Define S to be the set of the areas of all the distinct rectangles that can be inscribed in R. If the largest element of S can be written in the form $\frac{h\sqrt{k}}{l}$ where h and l are relatively prime positive integers and k is a squarefree positive integer, find $h + k + l$.

A) 59 B) 69 C) 79 D) 89 E) NOTA

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17) A Norman window is constructed by adjoining a semicircle to the top of a rectangular window such that one side of the rectangle is the diameter of the semicircle. If the outer perimeter of the window is 18 feet, find the maximum area of the window in feet squared.

A) $\frac{162}{\pi + 4}$ B) $\frac{162}{\pi + 2}$ C) $\frac{324}{\pi + 4}$ D) $\frac{324}{\pi + 2}$ E) NOTA

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18) Each of the 20 members of Mr. Frazer's class is given a distinct positive integer n in the interval $[1, 20]$ and is told to calculate $\int_{-\infty}^{\infty} \frac{\sin(nx)\sin(21x)}{x^2}\, dx$ If every member of the class correctly calculates their integral, find the sum of the values obtained.

A) 10π B) 11π C) 20π D) 21π E) NOTA

Hint: Product-to-Sum and conversion to a double integral

For Questions 19-20, you may use the following information. Daniel has an special ice cream cone that he is holding upright. The ice cream scoops on it can be represented by an infinite number of intersecting spheres whose centers are all colinear. Except for the first (bottom) scoop, each scoop has a radius $\frac{2}{3}$ that of the radius of the scoop under it, and $\frac{1}{4}$ of the diameter of each scoop intersects with the diameter of the scoop under it. The first scoop of Daniel's ice cream cone has radius 6 inches.

19) Find the height of the stack of ice cream scoops, in inches.

A) 24 B) 30 C) 32 D) 36 E) NOTA

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20) Find the volume of the stack of ice cream scoops, in inches cubed.

A) $\dfrac{1225\pi}{2}$ B) 758π C) $\dfrac{1675\pi}{2}$ D) 864π E) NOTA

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21) Buffy uses Simpson's approximation with 4 equally spaced subintervals to estimate the area bound by the curve $h(x) = x^3 + 6x^2 - 4x + 2$ and the x-axis on the interval $[1, 5]$. Given that Buffy gets the correct answer, find the value Buffy obtains.

A) 364 B) 366 C) 368 D) 370 E) NOTA

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22) A circular coin of diameter 2 inches is placed internally tangent to two sides of a square with side length 12 inches. The coin is slid clockwise around the perimeter of the square, always tangent to at least one side, until it is back where it started. Find the area of the square never covered by the coin in square inches.

A) $32 - 4\pi$ B) $64 - 4\pi$ C) $60 + \pi$ D) $68 - \pi$ E) NOTA

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23) Compute the finite area bound by the graph of $h(x) = 2x^3 - 6x^2 - 36x + 80$ and the x-axis.

A) $\dfrac{1715}{6}$ B) $\dfrac{1741}{6}$ C) $\dfrac{729}{2}$ D) $\dfrac{999}{2}$ E) NOTA

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24) Consider square $ABCD$ with side length 3. Points E, F, G, and H are chosen on sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ respectively to form a square. If E is chosen uniformly randomly along $\overline{AB}$, find the expected value of the area of $EFGH$.

A) $\dfrac{9}{2}$ B) 5 C) $\dfrac{16}{3}$ D) 6 E) NOTA

25) Consider triangle ZLU whose sides are changing with respect to time. ZL is growing at 3 units per second, LU is decreasing at 2 units per second, and ZU is decreasing at 1 unit per second. Find the rate that the inradius is changing at in units per second when $ZL = 13$, $LU = 14$, and $UZ = 15$.

A) $\frac{13}{210}$ B) $\frac{13}{112}$ C) $\frac{13}{84}$ D) $\frac{13}{28}$ E) NOTA

Hint: Twice the area of a triangle is equal to the product of its inradius and perimeter.

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26) Let R be the region in the first quadrant bounded by the graph of $x^4 + y^4 = xy(3 - 2xy)$. Compute the area of R.

A) $\frac{3}{16}$ B) $\frac{3}{8}$ C) $\frac{3}{4}$ D) $\frac{3}{2}$ E) NOTA

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27) Given a fixed positive perimeter, which of these shapes encloses more area than any of the others?

A) Equilateral Triangle B) Regular hexagon C) Regular nonagon

D) Square E) NOTA

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28) Compute the area of a square with diagonal length 28.

A) 196 B) 392 C) 784 D) 1568 E) NOTA

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29) Compute the total area of the part of the polar plane bounded by both $r = \cos(2\theta)$ and $r = \sin\theta$.

A) $\frac{\pi\sqrt{3}}{16}$ B) $\frac{3\pi\sqrt{3}}{16}$ C) $\frac{5\pi\sqrt{3}}{16}$ D) $\frac{3\pi\sqrt{3}}{8}$ E) NOTA

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30) Congratulations on making it to the end of the test!

A cube with side length 6 units shrinks uniformly so that its new side length is 5 units. Let A be the approximation for the new volume of the cube using a tangent line to $y = x^3$ at $x = 6$. Compute the positive difference between A and the new volume of the cube.

A) 0 B) 11 C) 17 D) 33 E) NOTA