What is the sum of the non-real roots to the equation $z^6 - 1 = 0$

State Convention 2021 Alpha Team Question 0

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The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -2 & 0 & 2 \end{bmatrix}$ has eigenvalues A, B, and C, where $A < B < C$, and determinant D.

Compute $B^A + C^D$.

State Convention 2021 Alpha Team Question 1

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Compute $B^A + C^D$.

State Convention 2021 Alpha Team Question 2

Find the sum of the solutions to each equation, then compute $\frac{\text{A}}{\pi} + 5\text{B}e^2 + \log_2\left(-\cos(C)\right)$

A) $\sin\frac{\theta}{4} + \cos\frac{\theta}{4} = 1,\ \theta \in [0, 16\pi)$

B) $\ln(t) + 2\ln(5) + \frac{[\ln(5)]^2}{\ln(t)} = \frac{4}{\ln(t)},\ t \in (0,1) \cup (1,\infty)$

C) $\sin\left(2^{2021}x\right) = \prod_{k=0}^{2020} \cos\left(2^k x\right),\ x \in \left(0, \frac{\pi}{2}\right]$

State Convention 2021 Alpha Team Question 2

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Let A be the eccentricity of the conic with equation $16x^2 - 9y^2 + 32x + 54y = 209$

Let B be the eccentricity of the conic with polar equation $r = \dfrac{7}{3 - \cos\theta}$

Let C be the smallest positive counter-clockwise angle of rotation needed to eliminate the xy term from the following conic: $(1+\sqrt{3})x^2 + xy + y^2 - 3x + 4y + 7 = 0$

Ellipse E has equation $16x^2 + 9y^2 + 32x + 54y = 47$, and point P lies on E along the major axis. Distinct points Q and R lie on ellipse E and share a y-value with each other, and the line $\overline{QR}$ passes through a focus of E. Let D be the difference between the maximum and minimum areas of triangle PQR.

Compute $\dfrac{ABD\pi}{C}$

State Convention 2021 Alpha Team Question 3

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Compute $\dfrac{ABD\pi}{C}$

State Convention 2021 Alpha Team Question 4

Let A be the number of distinguishable arrangements of the word HILLBILLY.

Let B be the probability that with three consecutive draws, one green and then two consecutive blue marbles are drawn from a bag containing 4 red marbles, 5 blue, and 3 green marbles. (No replacement!)

Let C be the number of trailing zeros at the end of 2020! when it is written in base 15.

Let D be the sum of the solutions to $(x+1)^{2020} + (x+2)^{2020} = 2020$

Compute $\mathrm{A} + \mathrm{B}^{-1} + \mathrm{C} + \mathrm{D}$

State Convention 2021 Alpha Team Question 4

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Let D be the sum of the solutions to $(x+1)^{2020} + (x+2)^{2020} = 2020$

Compute $\mathrm{A} + \mathrm{B}^{-1} + \mathrm{C} + \mathrm{D}$

$A = \cos\left(\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)$

$B = \sin\left(\cos^{-1}\left(\tan\left(\frac{7\pi}{4}\right)\right)\right)$

$C = \cot\left(\csc^{-1}\left(\frac{5}{3}\right)\right)$

$D = \cot^{-1}\left(\dfrac{2\tan\left(\frac{\pi}{12}\right)}{1-\tan^2\left(\frac{\pi}{12}\right)}\right)$

Find $A^2 + B^2 + C + \dfrac{D}{\pi}$.

State Convention 2021 Alpha Team Question 5

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Find $A^2 + B^2 + C + \dfrac{D}{\pi}$.

Given the functions $f(t) = \sin(t) + \cos(t)$, $g(t) = 2\sin(3t - 6\pi)$, and $h(t) = \sin(\sin(t))$,

Let A be the maximum value of $f(2t)$.

Let B be the amplitude divided by the period of $g(t)$.

Let C be the period of $h(t)$

Compute ABC.

State Convention 2021 Alpha Team Question 6

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Compute ABC.

Restrict the inverse trig functions to their traditional domains and ranges.

Let $[A, B]$ be the range of the function $y = -13\sin(3\pi x - \frac{\pi}{2}) + 22$

Let $[C, D]$ be the range of $y = \tan^{-1}(x+2) + \sin^{-1}(x+2) + 1$

Find $A + B + C + D$.

State Convention 2021 Alpha Team Question 7

Restrict the inverse trig functions to their traditional domains and ranges.

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Find $A + B + C + D$.

Let A be the distance between the polar points $\left(3, \frac{31\pi}{60}\right)$ and $\left(5, \frac{-9\pi}{60}\right)$.

Let B be the shortest distance between complex roots of the equation $(z-4+3i)^6 = 8$

Find $A^2 + B^2$.

State Convention 2021 Alpha Team Question 8

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Let B be the shortest distance between complex roots of the equation $(z-4+3i)^6 = 8$

Find $A^2 + B^2$.

For this question, consider the following vectors: $\vec{u} = \langle 1, 2, 3 \rangle$, $\vec{v} = \langle -8, 2, 1 \rangle$, $\vec{l} = \langle 0, 0, 7 \rangle$, and $\vec{m} = \langle -1, -2, 4 \rangle$

Let $A = \|\vec{u} \times \vec{l}\|$

Let $B = \|\vec{v} \times \vec{m}\|$

Let $C = \vec{l} \cdot (\vec{v} \times \vec{u})$

Let D be the $t-$value where the line $\vec{u}t + \vec{l}$ intersects the plane $\vec{v} \cdot \langle x, y, z \rangle = 1$

Find $\dfrac{A^2 + B^2}{5} + C + D$

State Convention 2021 Alpha Team Question 9

For this question, consider the following vectors: $\vec{u} = \langle 1, 2, 3 \rangle$, $\vec{v} = \langle -8, 2, 1 \rangle$, $\vec{l} = \langle 0, 0, 7 \rangle$, and $\vec{m} = \langle -1, -2, 4 \rangle$

Let $A = \|\vec{u} \times \vec{l}\|$

Let $B = \|\vec{v} \times \vec{m}\|$

Let $C = \vec{l} \cdot (\vec{v} \times \vec{u})$

Let D be the $t-$value where the line $\vec{u}t + \vec{l}$ intersects the plane $\vec{v} \cdot \langle x, y, z \rangle = 1$

Find $\dfrac{A^2 + B^2}{5} + C + D$

Let $\lfloor x \rfloor$ be the greatest integer that is less than or equal to x.

$$A = \lim_{t \to \infty} \frac{3t^2 + 4t + 5}{5t^2 + 6t + 7}$$

$$B = \lim_{x \to 3} \frac{x - 3}{\sqrt{|x^2 - 9|}}$$

$$C = \lim_{g \to e^2} \frac{1 + \ln(1 + \ln(g))}{\ln(e + \sqrt{g})}$$

$$D = \lim_{x \to \infty} \left(\sqrt[3]{x^3 - 3x^2 + 3x} - \sqrt{x^2 - 8x} \right)$$

Find the average of $A, B, \lfloor C \rfloor$, and D.

State Convention 2021 Alpha Team Question 10

Let $\lfloor x \rfloor$ be the greatest integer that is less than or equal to x.

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$$B = \lim_{x \to 3} \frac{x - 3}{\sqrt{|x^2 - 9|}}$$

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$$D = \lim_{x \to \infty} \left(\sqrt[3]{x^3 - 3x^2 + 3x} - \sqrt{x^2 - 8x} \right)$$

Find the average of $A, B, \lfloor C \rfloor$, and D.

$$A = \sum_{i=1}^{\infty} \frac{2i}{3(5^i)}$$

$$B = \sum_{j=0}^{\infty} \frac{\sin\left(\frac{j\pi}{2}\right)}{2^j}$$

For parts C and D, define $c_0 = 1$ and for $k > 0$, define $c_k = \dfrac{2}{c_{k-1}}$.

$$C = \prod_{k=0}^{\infty} c_k 2^{-k}$$

$$D = \sum_{k=0}^{\infty} c_k 2^{-k}$$

Find $ABCD$

State Convention 2021 Alpha Team Question 11

$$A = \sum_{i=1}^{\infty} \frac{2i}{3(5^i)}$$

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Find $ABCD$

State Convention 2021 Alpha Team Question 12

Given that $15xy = 4$, $3ab = 2$, $a + x = \frac{23}{45}$, and $b + y = \frac{20}{3}$ for rational a, b, c, d, find

$A = 50625x^4y^4 + 450x^2y^2$

$B = \frac{ab}{(a+x)(b+y)} + \frac{x}{a+x} + \frac{y}{b+y}$

$C = ay + bx$

$D = abxy$

Compute $A + 115B + 135C + 45D$

State Convention 2021 Alpha Team Question 12

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Compute $A + 115B + 135C + 45D$

State Convention 2021 Alpha Team Question 13

Solve the following and give your answer as the point (A, B, C, D)

$$\begin{aligned} 7 &= A + B + C + D \\ 4 &= 2A - B + 3C + D \\ -5 &= A - B - C - D \\ 29 &= 7A - 13B + 27C + 12D \end{aligned}$$

State Convention 2021 Alpha Team Question 13

Solve the following and give your answer as the point (A, B, C, D)

$$\begin{aligned} 7 &= A + B + C + D \\ 4 &= 2A - B + 3C + D \\ -5 &= A - B - C - D \\ 29 &= 7A - 13B + 27C + 12D \end{aligned}$$

Let $A < B$ be real solutions to $||x-4|+2| = 7$

Let $C < D$ be real solutions to $\ln|x-1| - \ln|4-2x| = \ln(7)$

Find $\dfrac{ABC}{D}$.

State Convention 2021 Alpha Team Question 14

Let $A < B$ be real solutions to $||x-4|+2| = 7$

Let $C < D$ be real solutions to $\ln|x-1| - \ln|4-2x| = \ln(7)$

Find $\dfrac{ABC}{D}$.