February Statewide Statistics Question: 1

For parts A-D, use the following set where X and Y are missing values. Answer each part independently of each other part.

{35, 41, 45, 50, 50, 58, 62, 68, 72, X, Y}

A: Suppose that the mean is 60, what is the value of X+Y?

B: What is the minimal value of the range of the data set?

C: Suppose that the data set is unimodal, what is the average of the data set's potential modes? Round to 3 decimal places.

D: Suppose that $35 < X \leq Y < 72, X \neq Y,$ and both X and Y are perfect squares. Find the average of the potential population standard deviations of the set. Round to 3 decimal places.

Find **A+B+C+D as a decimal**

February Statewide Statistics Question: 2

For parts A-D, fill in the blank with the appropriate statistical term and then calculate the number of distinct permutations of the letters in the word of that statistical term. For example, if the answer is CAT, then the answer for that part would be 6. Ignore any and all capitalizations.

A: In a(n) _______ random sample, the first person is selected at random from near the beginning of a list of all members of the population. Then, every kth person is sampled after that.

B: When observing data points in a scatterplot that is fit with a least squares regression line (LSRL), a(n) _____ data point in the scatterplot is any point that, if removed, changes the value of the correlation and/or the slope of the LSRL substantially. Such points are often outliers as well.

C: A graph known as a _____ shows two numeric values as ordered pairs for each bivariate observation, one corresponding to the value of the x-variable and one corresponding to the value of the y-variable.

D: The _______ of a data set is the sum of all the data divided by the number of entries in the data set.

Find $\frac{\boldsymbol{AC}}{\boldsymbol{BD}}$

February Statewide Statistics Question: 3

A: Given that a binomial distribution has n = 10 and p = 0.2 where n is the number of trials and p is the probability of success, find the sum of the expected value and the variance of the distribution.

B: Given that a binomial distribution has n = 10 and p = 0.2 where n is the number of trials and p is the probability of success, find the probability of obtaining exactly 7 failures in the 10 trials. Round to 1 decimal place.

C: Given that X is a geometric distribution has p = 0.2 where p is the probability of success, find the expected value of the square of the distribution. That is, find $E(X^2)$.

D: Given that a geometric distribution has p = 0.2 where p is the probability of success, find the probability of succeeding on the 3rd attempt.

Find **A+B+C+D**

February Statewide Statistics Question: 4

For parts A-D, use the following data.

{11, 13, 18, 19, 21, 22, 22, 22, 23, 24, 26, 28, 30, 45, 61}

A: Let the answer to this part equal 10 if the data set is skewed to the right. Let the answer to this part is equal to 100 if this data set is skewed to the left. If neither are true, let the answer to this part equal to 1000.

B: Find the product of the upper fence and lower fence when using the 1.5IQR rule. In other words, find the product of the maximum and minimum value a data value can take without being considered an outlier using the 1.5IQR rule.

C: Find the sum of the outliers of the data set using the 1.5IQR rule. If there are no outliers, the answer to this part is 0. If there is only one outlier, that value is the answer to this part.

D: Find the sum of the numbers in the 5-number summary of the data set.

Find **A+B+C+D**

February Statewide Statistics Question: 5

There is a total of 120 students in the MAO chapter at DBHS. They can take Spanish, French, or Japanese language classes. 60 students take Spanish, 50 take French, and 20 take Japanese. 12 take Spanish and French only, 4 take French and Japanese only, and 2 take Spanish and Japanese only. Only 2 students take all 3 language classes. **Answer each part as a fraction.**

A: What is the probability of selecting a student that does not take French, Japanese, or Spanish?

B: What is the probability of selecting a student that only takes exactly 1 language class?

C: What is the probability of selecting a student that takes multiple language classes given that they take at least 1 language class?

D: What is the probability of selecting a student that takes Japanese given that they take Spanish?

Find **A+B+C+D as a fraction**

February Statewide Statistics Question: 6

Mr. Snow decides to randomly poll a SRS of 41 out of the thousands of students from his morning classes over the years and a SRS of 35 students out of the thousands of students from his afternoon classes over the years and asks his students if they think that he is being an effective teacher. For his morning classes, 32 out of the 41 students said he is being an effective teacher and for his afternoon classes 18 out of the 35 students said he is being an effective teacher. Mr. Snow believes that there is a difference between the perceptions of his teaching efficacy for his morning classes and afternoon classes over the years. You may assume all conditions are met for all parts.

A: Find the sum of the p-value and the absolute value of the test statistic of the 2 proportion Z-test that can be used to calculate the perceptions of Mr. Snow's teaching efficacy. Round to two decimals.

B: Suppose Mr. Snow is conducting the appropriate test at the 5% significance level. Let the answer to this part be equal to 10 if he rejects the null. Let the answer to this part be equal to 100 if he fails to reject the null. If neither are true, let the answer to this part be equal to 1000.

C: How many tails are included for the appropriate test?

D: Suppose Mr. Snow constructs a 95% confidence interval for the difference between the two parameters of interest in order to see if he can reject the same null hypothesis from above. He calculates the interval to be (a, b) where $|a| \leq |b|$**.** Give the value of a if he rejects the null. Give the value of b if he fails to reject the null. If neither are true, give the value of ab. Round to two decimals.

Find **ABCD**

February Statewide Statistics Question: 7

Die X is a fair 6-sided die labeled from 1-6. Die Y is an unfair 6-sided die labeled from 1-6 where the probability of rolling a number k is twice as likely than rolling a number $k - 1$.

A: Find the value of $E(X) + E(Y) + E(XY)$.

B: Find $P(X \geq Y - 1)$.

C: Find $P(X = Y)$.

D: Suppose a die is chosen at random, what is the probability that the die rolled shows a prime number?

Find **A+B+C+D**

February Statewide Statistics Question: 8

Clownboy is suspicious that the amount of Fritos chips in an average Frito bag is actually less than advertised by the Fritos company. The Fritos company advertises that there is a mean of 160 grams worth of Fritos chips in a bag. Clownboy decides to take an SRS of 144 Frito bags and finds that the mean grams were 155 with standard deviation 36 grams.

A: Find the sum of the p-value and test statistic of the appropriate test. Round to 2 decimals.

B: Suppose Clownboy is testing at the 1% significance level. Let the answer to this part equal 10 if he rejects the null. Let the answer to this part equal 100 if he fails to reject the null. If neither are true, let the answer to this part equal 1000.

C: How many tails are included in the appropriate test?

D: What is the value of the standard error used to calculate the appropriate test statistic? Round to 2 decimals.

Find **ABCD**

February Statewide Statistics Question: 9

For parts A-D, there is a bag with 5 white marbles, 5 blue marbles, and 5 red marbles. Vedant picks a marble at random, then notes its color, and then put back into the bag.

A: What is the probability of randomly selecting 3 red marbles in a row?

B: What is the probability of randomly selecting a white marble, a blue marble, and a red marble in any order?

C: What is the probability of selecting 5 marbles and none of them are red?

D: Suppose that k purple marbles are added into the bag and suppose that Vedant will continue randomly selecting a marble (again, with replacement) until he notes a purple marble. He calculates that the average number of times he has to randomly select a marble until he notes a purple marble is $\frac{5}{2}$. Find the value of k.

Find $\frac{CD}{AB}$

February Statewide Statistics Question: 10

Anna is curious about the linear relationship between a student's grade in Mr. Snow's statistics class and how many hours they play League of Legends in a given week. She randomly asks 11 students and finds the following (x,y) pairs where x is the student's grade, and y is the student's hours played in a given week.

Data: (82, 0), (71, 1), (87, 1), (75, 2), (85, 4), (89, 6), (87, 6), (90, 7), (92, 8), (98, 12), (100, 24)

A: Without actually creating a LSRL, the point (a, b) must lie on the LSRL. Find the value of $a + b$.

B: Find the sum of the slope, coefficient of determination, and the y-intercept of the LSRL. Round to 2 decimals.

C: Thiago, who was not polled by Anna, has a 79 in Mr. Snow's statistics class. Based on the LSRL where the slope and y-intercept are rounded to 2 decimals, what is his predicted number of weekly hours of League of Legends that he would play? Round to the nearest hour.

D: Find the residual of the point (100,24) based on the LSRL where the slope and y-intercept are rounded to 2 decimals.

Find **A+B+C+D**

February Statewide Statistics Question: 11

Mr. Snow is testing how Frito chip consumption affects scores in MAO competitions. He predicts that eating Fritos will increase scores and that eating even more Fritos will increase scores even more. He decides to randomly assign 30 DBHS competitors equally to the following groups: {1 bag of Fritos, 3 bags of Fritos, 5 bags of Fritos, 10 bags of Fritos, No Fritos}.

A: The number of factors and levels in Mr. Snow's experiment can be written as α and δ respectively. Find the value of $2\alpha + 3\beta$.

B: How many participants are given each treatment?

C: Suppose that the mean MAO score for those that ate 1 bag Classic Fritos is 65 with standard deviation 10 and that those that ate no Fritos is 55 with standard deviation 6. The p-value of the appropriate test statistic rounded to 2 decimals can be written as x. If Mr. Snow rejects the null at the 5% significance level, find x. Otherwise, find $2x$. Assume all conditions are met and you must use your calculator program and the "exact" degrees of freedom to perform the test.

D: Now suppose that the mean MAO score for those that ate 10 bags of Classic Fritos is a 61 with standard deviation 8. Comparing this with the 1 bag Classic Fritos data in part C, the p-value of the appropriate test statistic rounded to 2 decimals can be written as y. Mr. Snow rejects the null at the 5% significance level, find y. Otherwise, find $2y$. Assume all conditions are met and you must use your calculator program and the "exact" degrees of freedom to perform the test.

Find **A+B+C+D**

February Statewide Statistics Question: 12

Ms. Lambert loves geometry. In fact, she loves it so much that she has a semi-circular pdf (probability density function) over the domain [0,x].

A: Find the exact value of x.

B: Find the exact value of the median.

C: Find the exact value of the mean.

D: What is the exact value of the mode of the pdf?

Find $(\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{D})\boldsymbol{C\pi}$

February Statewide — Statistics Question: 13

Mr. Otto has just given a calculus exam to his students. The mean of the scores is a 78 with variance 36. Mr. Otto decides to curve the exam by altering the standard deviation to 9 and then giving every student an extra 5 points. Assume that the scores follow a normal distribution and that students can get a score over 100.

A: The transformation equation can be written in the form $y = \alpha x + \beta$. Find the value of the expression $2\alpha + 3\beta$.

B: Maverick got a 92 before the curve. Let the value of his score after the curve rounded to the nearest integer be equal to δ. Oshmita got an 81 after the curve. Let the value of her score before the curve was applied rounded to the nearest integer ε. Find $2\delta - 3\varepsilon$.

C: Find the value of the mean of the scores of the calculus exams after the curve.

D: Suppose that Mr. Otto wants the mean scores of the calculus exams after the curve to be an 85 with the same standard deviation of 9. So, instead of adding the 5 extra points he adds k extra points to each calculus exam. Find the value of $2k$.

Find **A+B+C+D**

February Statewide — Statistics Question: 14

Mr. Snow has just given an exam where the mean is an 82 with standard deviation 9. Assume that the distribution of the exam scores are approximately normal.

A: Shiv scored a 100 on the exam. Find the value of his z-score.

B: Austin knows that his z-score is $\frac{7}{9}$. Find the value of his raw score.

C: Khushi scored at the 90th percentile. Find the value of her raw score. Round to the nearest integer.

D: Duncan scored a 97. Calculate his percentile to the nearest percent. For example, if he scored at the 20th percentile, the answer to this part is 20.

Find **A+B+C+D**

For parts A-D, use the following stem and leaf plot:

Stem	Leaf
2	0 0 2 3 4 6 7 9
3	0 1 1 1 2 5 7 7 8 9 9
4	2 3 5 6 9
5	1 1 2 7 9 9

Where $2|0$ represents 20.

A: Find the value of the mean of the distribution.

B: Find the value of the standard deviation of the distribution. Round to 2 decimals.

C: Find the mode of the distribution.

D: Find the median of the distribution.

Find $100(\boldsymbol{A}+\boldsymbol{B}+\boldsymbol{C}+\boldsymbol{D})$

Answers:

1.) 282.596

2.) 50400

3.) 48.928 or $\frac{6116}{125}$

4.) 485.25 or $\frac{1941}{4}$

5.) $\frac{293}{270}$

6.) 2.94 or $\frac{147}{50}$

7.) $\frac{20791}{756}$

8.) -486

9.) 160

10.) $\frac{63397}{1100}$ or $57.63\overline{36}$

11.) 24.57 or $\frac{2457}{100}$

12.) 8

13.) 209

14.) 280

15.) 11721

Solutions:

1.) 282.596

A = 179, the mean is the sum of numbers divided by the number of numbers. Thus, we get

$60 = \frac{35+41+45+50+50+58+62+68+72+X+Y}{11}$

$660 = 481 + X + Y$

$179 = X + Y$

B = 37, the minimal range of the data happens when 37 is the minimum and where 72 is the maximum and therefore $35 < X \leq Y < 72$. Thus, the range is equal to 72 – 35 = 37

C = 54.875, since the data set is unimodal, we know that there can only be one mode depending on the value of X and Y. We see that if both X and Y are 37 then the mode would be 37, this pattern continues for each value that is written in our data set. Suppose that X and Y are equal to each other but are not one of the other points we would have two modes of 50 and whatever value of X and Y are which makes the statement that the data set is unimodal false. Therefore, our potential modes are: 35, 41, 45, 50, 58, 62, 68, 72. Computing the average:

$$\frac{35+41+45+50+58+62+68+72}{8} = \frac{431}{8} = 54.875$$

D = 11.721, since both X and Y are between 35 and 71 and aren't equal to each other and are perfect squares we only have the potential values for X and Y: 36, 49, and 64. Since order doesn't matter we will have a total of 3C2 = $\frac{3!}{2!\cdot 1!} = \frac{3\cdot 2\cdot 1}{2\cdot 1\cdot 1} = 3$ different data sets. One with 36 and 49, one with 36 and 64 and one with 49 and 64. Plugging these into L1, L2, and L3 then running 1Var Stats for each we get the following population standard deviations respectively:

11.75788816 (when 36 and 49 are plugged in)

12.25352032 (when 36 and 64 are plugged in)

11.15184453 (when 49 and 64 are plugged in)

Now, finding the average we get: $\frac{11.75788816+12.25352032+11.15184453}{3} = \frac{35.16324301}{3} = 11.721081 = 11.721$

A+B+C+D = 179+37+54.875+11.721 = 282.596

2.) 50400

A = 907200, the word is systematic. The number of the distinct permutations of the letters in the word is equal to $\frac{10!}{2!\cdot 2!} = 907200$ as there are a total of 10 letters with 2 repeated S's and 2 repeated T's.

B = 4989600, the word is influential. The number of the distinct permutations of the letters in the word is equal to $\frac{11!}{2!\cdot 2!\cdot 2!} = 4989600$ as there are a total of 10 letters with 2 repeated I's, 2 repeated N's, and 2 repeated L's.

C = 6652800, the word is scatterplot. The number of the distinct permutations of the letters in the word is equal to $\frac{11!}{3!} = 6652800$ as there are a total of 10 letters with 3 repeated T's.

D = 24, the word is mean. The number of the distinct permutations of the letters in the word is equal to $4! = 24$ as there are a total of 4 letters.

$$\frac{AC}{BD} = \frac{(907200)(6652800)}{(4989600)(24)} = 50400$$

3.) 48.928 or $\frac{6116}{125}$

A = 3.6, the expected value of a binomial distribution is np. In our case np = 10(.2) = 2. The variance is equal to np(1-p) = 10(.2)(.8) = 1.6. 2+1.6 = 3.6

B = .2, this is a binomial probability with p = .2, n = 10, and the amount of successes is 3. Thus, we plug in the calculator binompdf(10, .2, 3) - .201326592 = .2

C = 45, since we are asked to find the expected value of the square of the distribution we are asked to find $E(X^2)$. To find this value we use the formula $E(X^2) = E(X)^2 + Var(X)$. Thus,

$E(X) = \frac{1}{p} = \frac{1}{.2} = 5$

$Var(X) = \frac{1-p}{p^2} = \frac{.8}{.04} = 20$

Thus,

$E(X^2) = 5^2 + 20 = 25 + 20 = 45$

D = .128, we are looking for succeeding on the 3rd attempt which means that we failed the first 2. Thus, $(1-p)^2(p) = (.8)^2(.2) = .128$

A+B+C+D = 3.6+.2+45+.128 = 48.928 or $\frac{6116}{125}$

4.) 485.25 or $\frac{1941}{4}$

A = 10, the statement that the distribution is skewed to the right is true. There are plenty of ways to verify this such as creating a boxplot, stem plot, or a histogram.

B = 228.25 or $\frac{913}{4}$, using the 1.5IQR rule we must first find the IQR. After plugging in the data into L1 and running 1Var Stats we find that Q3 = 28 and Q1 = 19. Thus, the IQR = 28 – 19 = 9. Multiplying this by 1.5 we get 13.5. Adding this value to Q3 and subtracting this value by Q1 we get the values 41.5 and 5.5 respectively meaning that within the closed interval [5.5,41.5] the data is not considered an outlier, that is, 5.5 and 41.5 are our lower and upper fence respectively. Thus, the product = (5.5)(41.5) = 228.25 = $\frac{913}{4}$.

C = 106, using the information from part B we find that the lower and upper fences are 5.5 and 41.5. We can clearly see that both 45 and 61 are outliers. The sum of which is equal to 45+61 = 106

D = 141, the 5-number summary of a data set includes the min, Q1, median, Q3, and max. We can find these using 1Var Stats. Thus,

min = 11

Q1 = 19

med = 22

Q3 = 28

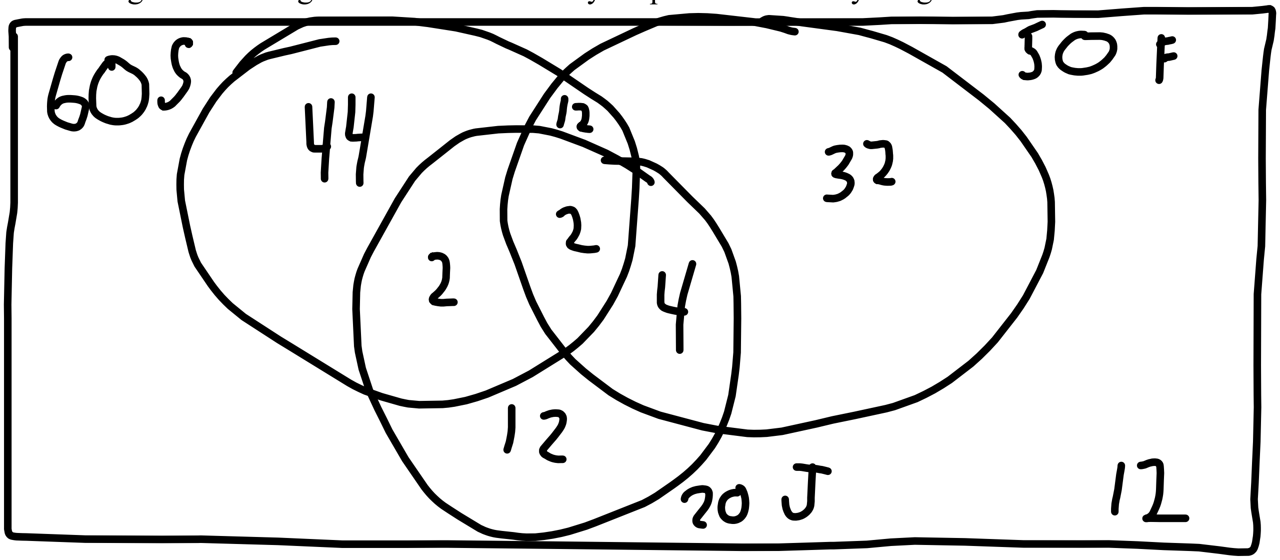
max = 61

The sum of these numbers is equal to 11+19+22+28+61 = 141

A+B+C+D = $10 + \left(\frac{913}{4}\right) + 106 + 141$ = 485.25 or $\frac{1941}{4}$

5.) $\frac{293}{270}$

Constructing a Venn Diagram will tremendously help visualize everything.



A = $\frac{1}{10}$, as seen in the diagram there are only 12 students who do not take any of these language classes. Thus, the probability is $\frac{12}{120} = \frac{1}{10}$

B = $\frac{11}{15}$, as seen in the diagram only 44 take only Spanish, only 32 take only French, and only 12 take only Japanese. The sum of which is 88. Thus, the probability is $\frac{88}{120} = \frac{11}{15}$.

C = $\frac{5}{27}$, as seen in the diagram there are a total of 120 – 12 = 108 students that take at least one language. The number of students that take multiple language classes is equal to 2+12+4+2 = 20. Thus, the probability is $\frac{20}{108} = \frac{5}{27}$.

D = $\frac{1}{15}$, there are a total of 60 Spanish students and the overlap of the number of students between Japanese and Spanish is equal to 2+2 = 4. Thus, the probability is $\frac{4}{60} = \frac{1}{15}$.

A+B+C+D = $\frac{1}{10} + \frac{11}{15} + \frac{5}{27} + \frac{1}{15} = \frac{293}{270}$

6.) 2.94 or $\frac{147}{50}$

A =2.45, this scenario is a 2 proportion Z-test with the following:

x1: 32

n1: 41

x2: 18

n2: 35

p1 ≠ p2 (as Mr. Snow is testing if there is a difference and therefore the test is two-tailed)

p = .0147598727

z = 2.438217996

p+z = . .0147598727 + 2.438217996 = 2.452977869 = 2.45 when rounded

B = 10, since we have found the p-value above and since that the alpha value (.05) > p-value (.01) we reject the null hypothesis. Note, the answer of 1000 never happens since we either reject the null or we fail to reject the null, there is no in between.

C = 2, as mentioned in part A, Mr. Snow is testing if there is a difference and therefore the test is two-tailed.

D = .06, this scenario is a 2 proportion Z-interval with the following:

x1: 32

n1: 41

x2: 18

n2: 35

C-Level: .95

We get the confidence interval (.05771, .47469). Since 0 is not in the interval, then Mr. Snow rejects the null hypothesis. Therefore, $a = .05771$ = .06 when rounded

Note, if a student swaps the values of x1 with x2 and n1 with n2 they get a confidence interval where the statement $|a| \leq |b|$ is not true. Specifically, they would get (-.4747, -.0577). Clearly, $.4747$ is not less than or equal to $.0577$. Therefore, any disputes about ordering should be disregarded entirely.

ABCD = (2.45)(10)(2)(.06) = 2.94 or $\frac{147}{50}$

7.) $\mathbf{\frac{20791}{756}}$

Constructing the probabilities for each number for each die is crucial for all the parts.

Constructing Die X is quite simple as each value is equal with probability $\frac{1}{6}$.

Constructing Die Y is trickier. Since the probability of rolling k is twice as likely as k-1 we can construct every probability in terms of a variable. In our case, let's use w. So, $P(Y=6)=w, P(Y=5)=\frac{w}{2}, P(Y=4)=\frac{w}{4}$ ... clearly we see that we find this geometric sum. We find that the sum is equal to $w+\frac{w}{2}+\frac{w}{4}+\frac{w}{8}+\frac{w}{16}+\frac{w}{32}=\frac{63w}{32}$. Due to the rules of probability, the sum must equal to one. Thus,

$$\frac{63w}{32}=1$$

$$w=32/63$$

So, writing our probabilities for Y = 1, Y = 2 … Y=6. We get the following: $\frac{1}{63},\frac{2}{63},\frac{4}{63},\frac{8}{63},\frac{16}{63},\frac{32}{63}$. Now we are ready to do some calculations.

A =$\mathbf{\frac{185}{7}}$, to find the expected value we multiply the value given on the die by the probability it occurs. Thus,

$$E(X)=1\left(\frac{1}{6}\right)+2\left(\frac{1}{6}\right)+3\left(\frac{1}{6}\right)+4\left(\frac{1}{6}\right)+5\left(\frac{1}{6}\right)+6\left(\frac{1}{6}\right)=\frac{21}{6}=\frac{7}{2}$$

$$E(Y)=1\left(\frac{1}{63}\right)+2\left(\frac{2}{63}\right)+3\left(\frac{4}{63}\right)+4\left(\frac{8}{63}\right)+5\left(\frac{16}{63}\right)+6\left(\frac{32}{63}\right)=\frac{321}{63}=\frac{107}{21}$$

$$E(XY)=E(X)E(Y)$$

**(this is true, similar to Venn Diagrams, since the distributions of X and Y are independent).

$$E(XY)=\left(\frac{7}{2}\right)\left(\frac{107}{21}\right)=\frac{107}{6}$$

$$E(X)+E(Y)+E(XY)=\frac{7}{2}+\frac{107}{21}+\frac{107}{6}=\frac{185}{7}$$

B =$\mathbf{\frac{13}{27}}$, To solve this, we have to look at things on a case by case basis. The question is asking for $P(X\geq Y-1)=P(X\geq 0 \text{ and } Y=1)+P(X\geq 1 \text{ and } Y=2)+P(X\geq 2 \text{ and } Y=3)+P(X\geq 3 \text{ and } Y=4)+P(X\geq 4 \text{ and } Y=5)+P(X\geq 5 \text{ and } Y=6)$. Now, here are the six individual terms which are easily obtained because the two dice are clearly independent of each other:

$$P(X\geq 0\ \cap Y=1)=\left(\frac{6}{6}\right)\left(\frac{1}{63}\right)=\frac{6}{378}=\frac{1}{63}$$

$$P(X\geq 1\ \cap Y=2)=\left(\frac{6}{6}\right)\left(\frac{2}{63}\right)=\frac{12}{378}=\frac{2}{63}$$

$P(X \geq 2\ \cap Y = 3) = \left(\frac{5}{6}\right)\left(\frac{4}{63}\right) = \frac{20}{378} = \frac{10}{189}$

$P(X \geq 3\ \cap Y = 4) = \left(\frac{4}{6}\right)\left(\frac{8}{63}\right) = \frac{32}{378} = \frac{16}{189}$

$P(X \geq 4\ \cap Y = 5) = \left(\frac{3}{6}\right)\left(\frac{16}{63}\right) = \frac{48}{378} = \frac{8}{63}$

$P(X \geq 5\ \cap Y = 6) = \left(\frac{2}{6}\right)\left(\frac{32}{63}\right) = \frac{64}{378} = \frac{32}{189}$

Now summing up the above values using the common denominator of 378 we get:

$P(X \geq Y - 1) = \left(\frac{6+12+20+32+48+64}{378}\right) = \frac{182}{378} = \frac{13}{27}$

C = $\boldsymbol{\frac{1}{6}}$**,** to find this we sum for each number from 1-6. Thus,

$P(X = 1) \cdot P(Y = 1) = \left(\frac{1}{6}\right)\left(\frac{1}{63}\right) = \frac{1}{378}$

$P(X = 2) \cdot P(Y = 2) = \left(\frac{1}{6}\right)\left(\frac{2}{63}\right) = \frac{2}{378}$

$P(X = 3) \cdot P(Y = 3) = \left(\frac{1}{6}\right)\left(\frac{4}{63}\right) = \frac{4}{378}$

$P(X = 4) \cdot P(Y = 4) = \left(\frac{1}{6}\right)\left(\frac{8}{63}\right) = \frac{8}{378}$

$P(X = 5) \cdot P(Y = 5) = \left(\frac{1}{6}\right)\left(\frac{16}{63}\right) = \frac{16}{378}$

$P(X = 6) \cdot P(Y = 6) = \left(\frac{1}{6}\right)\left(\frac{32}{63}\right) = \frac{32}{378}$

The sum of which is $\frac{63}{378} = \frac{1}{6}$.

Without doing any calculations, we can think about this theoretically/conceptually. Since the first die is consistently $\frac{1}{6}$, the overall probability will be $\frac{1}{6}$ no matter how the other die varies. Thinking about this idea in a more intuitive fashion, imagine a fair coin and a coin that only lands heads. The probability of the coins matching is where the fair coin is heads, which is $\frac{1}{2}$.

D = $\boldsymbol{\frac{107}{252}}$**,** prime numbers are 2,3,5 and since we are choosing a die at random, we multiply each die's probability of prime by $\frac{1}{2}$ then sum them together. Thus,

$P(X = prime) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$
$P(Y = prime) = \frac{2}{63} + \frac{4}{63} + \frac{16}{63} = \frac{22}{63} = \frac{22}{63}$

$P(randomly\ choosing\ X\ or\ Y = prime) = \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{22}{63}\right) = \frac{1}{4} + \frac{11}{63} = \frac{107}{252}$

A+B+C+D = $\frac{185}{7} + \frac{13}{27} + \frac{1}{6} + \frac{107}{252} = \frac{20791}{756}$

8.) -486

For parts A-D, we realize that the test is a T-Test since the population standard deviation (i.e. the standard deviation for all Fritos is not known, only thing we know is Clownboy's sample standard deviation. Thus, we use the following for our T-Test:

$\mu_0 = 160$

$\bar{x}: 155$

$Sx: 36$

$n: 144$

$\mu < \mu_0$ (Since Clownboy expects that the actual number of grams of Frito chips is less than what the company advertises).

We get the following:

t = -1.66666667

p = .0488847591

$\bar{x}: 155$

$Sx: 36$

$n: 144$

A= -1.62, we are asked to find the p-value and test statistic which is p and t respectively. The sum of both is $p + t = .0488847591 + -1.66666667 = -1.617781908 = -1.62$ when rounded

B= 100, since the alpha value (.01) is <u>not greater</u> than the p-value (.0488847591) we fail to reject the null hypothesis meaning that the answer to this part is 100.

C= 1, Since Clownboy expects that the actual number of grams of Frito chips is less than what the company advertises he is calculating only 1 tail since he is only concerned about values being less than what the company advertises.

D= 3, standard error, also commonly written as SE, is equal to the standard deviation divided by the square root of n. Thus,

$SE = \frac{36}{\sqrt{144}} = \frac{36}{12} = 3$

ABCD = (-1.62)(100)(1)(3) = -486

9.) 160

A = $\boldsymbol{\frac{1}{27}}$ $\boldsymbol{or\ .\overline{037},}$ since the probability of red is $\frac{5}{15} = \frac{1}{3}$. We repeat this 3 times since each trial will be independent of each other. Thus, $P(Red\ 3\ times) = \left(\frac{1}{3}\right)^3 = \frac{1}{27} = .\overline{037}$

B = $\boldsymbol{\frac{2}{9}}$ $\boldsymbol{or\ .\overline{2}}$ **,** selecting one of each is equal to $\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{27}$. But we must account for the fact that we can do it in any order meaning that we have count all the ways of getting WRB = 3! = 6. Thus, we multiply this with our $\frac{1}{27}$ getting $6\left(\frac{1}{27}\right) = \frac{2}{9}\ or\ .\overline{2}$

C = $\boldsymbol{\frac{32}{243}}$**,** we want to find the probability of not selecting red, then repeating this step 5 times. Thus, $P(no\ red\ after\ 5\ tries) = \left(1 - \frac{1}{3}\right)^5 = \left(\frac{2}{3}\right)^5 = \frac{32}{243}$

D = 10, since we have k marbles than the probability of selecting a purple marble is equal to $\frac{k}{k+15}$. In other words, $P(purple) = \frac{k}{k+15}$. Now, to find the expected value (in this case of a geometric distribution, we use $\frac{1}{p}$. Thus,

$$E(x) = \frac{1}{\frac{k}{k+15}}$$

$$\frac{5}{2} = \frac{k+15}{k}$$

$$5k = 2k + 30$$

$$3k = 30$$

$$k = 10$$

$$\boldsymbol{\frac{CD}{AB} = \frac{\left(\frac{32}{243}\right)(10)}{\left(\frac{1}{27}\right)\left(\frac{2}{9}\right)} = 160}$$

10.) $\boldsymbol{\frac{63397}{1100}}$ **or** $\boldsymbol{57.63\overline{36}}$

A = $\boldsymbol{\frac{1027}{11}}$ **or** $\boldsymbol{93.\overline{36},}$ the mean of the x values and the mean of the y values must lie on the LSRL, also commonly written as $(\bar{x}, \bar{y})$. Plugging in the x values into L1 and the y values into L2 and running 1 Var stats for both we find that

$$a = \frac{956}{11}$$

$$b = \frac{71}{11}$$

$$a + b = \frac{956}{11} + \frac{71}{11} = \frac{1027}{11} = 93.\overline{36}$$

Note, although the inclusion of $93.\overline{36}$ is mathematically awkward, it still has the same value as the fraction. Therefore, either are acceptable.

B = -46.87, with the values already in L1 and L2 we calculate the LSRL in LinReg(ax+b) we get:

$y = ax + b$

$a = 0.6281527744$

$b = -48.13764112$

$r^2 = 0.6371681556$

$r = 0.7982281351$

Clearly, we see that the slope, coefficient of determination and y-intercept are a, r^2, and b respectively. The sum of which is:

$0.6281527744 + 0.6371681556 + -48.13764112 = -46.87232019 = -46.87$

C = 2, given that the LSRL's slope and y-intercept are both rounded to 2 decimal places we get the following:

$y = .63x - 48.14$

Now, we plug in 79 (as this is Thiago's grade in Mr. Snow's class) to find his predicted weekly hours of playing League of Legends. Thus,

$y = .63(79) - 48.14 = 49.77 - 48.14 = 1.63 = 2$ when rounded

D = 9.14,

$residual = observed - predicted$

$residual = 24 - (.63(100) - 48.14)$

$residual = 24 - (63 - 48.14)$

$residual = 24 - 14.86$

$residual = 9.14$

A+B+C+D = $\left(\frac{1027}{11}\right) + (-46.87) + (2) + (9.14) = \frac{63397}{1100}$ **or** $57.63\overline{36}$

11.) 24.57 or $\frac{2457}{100}$

A = 17, there is only 1 factor, eating Fritos. There is a total of 5 levels, eating (0,1,3,5, or 10) bags of Fritos. So,

$\alpha = 1$

$\beta = 5$

$2\alpha + 3\beta = 2(1) + 3(5) = 17$

B = 6, since there is a total of 5 different types of treatments then we divide the total number by these different types. Thus, $\frac{30}{5} = 6$.

C = .03 or $\frac{3}{100}$, this is a 2 sample T-Test (since these are sample standard deviations) with

$\bar{x}1: 65$

$Sx1: 10$

$n1: 6$

$\bar{x}2: 55$

$Sx2: 6$

$n2: 6$

$\mu_1 > \mu_2$ (Since Mr. Snow believes that eating Fritos will increase scores compared to not eating Fritos)

We get the following p-value: .0340530881 = .03 when rounded. Thus, $x = .05$. At the 5% significance level we would reject the null since the alpha level (.05) is greater than the p-value. Thus, Mr. Snow rejects the null hypothesis and therefore the answer to this part is simply $x = .03$

D = $\mathbf{1.54}$ $\boldsymbol{or}$ $\frac{77}{50}$, same process described in part C.

$\bar{x}1: 65$

$Sx1: 10$

$n1: 6$

$\bar{x}2: 61$

$Sx2: 8$

$n2: 6$

$\mu_1 < \mu_2$ (Since Mr. Snow believes that eating more Fritos will increase scores compared to eating less Fritos i.e. eating 1 Frito bag will have a less of an increase than 10 Frito bags).

We get a p-value of .7650920557 = .77 when rounded. So, $y = .77$. Since the alpha level (.05) is not greater than the p-value, Mr. Snow fails to reject the null. Therefore, we calculate $2y = 2(.77) = 1.54$ or $\frac{77}{50}$.

A+B+C+D = 17+6+(.03)+(1.54) = 24.57 or $\frac{2457}{100}$

12.) 8

A = $\frac{2\sqrt{2\pi}}{\pi}$, in order to be a pdf then the area must be equal to 1. The area of a circle is πr^2**.** So the area of a semicircle is $\frac{\pi r^2}{2}$**.** We now set this equal to 1. Thus,

$1 = \frac{\pi r^2}{2}$

$\frac{2}{\pi} = r^2$

$r = \sqrt{\frac{2}{\pi}}$

$r = \frac{\sqrt{2\pi}}{\pi}$

This is the distance between 0 and the center of the circle, we now multiply by 2 to get the endpoint. Therefore, the domain of our semicircular pdf is $\left[0, \frac{2\sqrt{2\pi}}{\pi}\right]$ thus, $x = \frac{2\sqrt{2\pi}}{\pi}$

B = C = D = $\frac{\sqrt{2\pi}}{\pi}$, since the pdf in question is a perfectly symmetric semi-circle centered at $\left(\frac{\sqrt{2\pi}}{\pi}, 0\right)$, we have the following property: mean = median = mode.

$(\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{D})\boldsymbol{C}\boldsymbol{\pi} = \left(\frac{2\sqrt{2\pi}}{\pi} + \frac{\sqrt{2\pi}}{\pi} + \frac{\sqrt{2\pi}}{\pi}\right)\left(\frac{\sqrt{2\pi}}{\pi}\boldsymbol{\pi}\right) = \frac{4\pi+2\pi+2\pi}{\pi} = \frac{8\pi}{\pi} = \mathbf{8}$

13.) 209

A = 18, in order to create the transformation equation, we have to first find the value to multiply both the standard deviation and the mean by, then we can add the 5 extra points last. Since the variance for the exam before the curve is 36, then the standard deviation is $\sqrt{36}$ = 6. Now, in order to get a new standard deviation of 9 we need to multiply by $\frac{3}{2}$ (this is our value for the slope of the transformation equation). Now, we add the 5 points to every score last (this is the y-intercept of the transformation equation). Thus,

$y = \frac{3}{2}x + 5$

$\alpha = \frac{3}{2}$

$\beta = 5$

$2\alpha + 3\beta = 2\left(\frac{3}{2}\right) + 3(5) = 3 + 15 = 18$

B = 133, since Maverick got a 92 before the curve, we just plug it into the transformation that we obtain from part A. Thus,

$y = \frac{3}{2}(92) + 5 = 138 + 5 = 143$

$\delta = 143$

$81 = \frac{3}{2}x + 5$

$76 = \frac{3}{2}x$

$\frac{152}{3} = x$

$x = 50.\overline{6}$ which rounds to 51

$\varepsilon = 51$

$2\delta - 3\varepsilon = 2(143) - 3(51) = 286 - 153 = 133$

C = 122, since the mean was a 78 before the curve, we can just plug it into the transformation equation to find the new mean for the distribution post-curve. Thus,

$y = \frac{3}{2}(78) + 5 = 117 + 5 = 122$

D = -64, so, we need to alter our transformation equation in the following manner:

$y = \frac{3}{2}x + k$ (we keep the $\frac{3}{2}$ since the standard deviation remains the same)

Now, similar to part C, we need to plug in the old mean for x, and plug in the new mean (85) for y, since this is what we want in the end, and all we have to solve for k. Thus,

$85 = \frac{3}{2}(78) + k$

$85 = 117 + k$

$k = -32$

$2k = 2(-32) = -64$

A+B+C+D = 18+133+122+-64 = 209

14.) 280

A = 2,

$z = \frac{x - \mu}{\sigma} = \frac{100-82}{9} = \frac{18}{9} = 2$

B = 89,

$z = \frac{x - \mu}{\sigma}$

$x = z\sigma + \mu = \left(\frac{7}{9}\right)(9) + 82 = 7 + 82 = 89$

C = 94, we need to use InvNorm to calculate Khushi's z-score. We plug the following:

area: .9

μ: 82

σ: 9

LEFT (since 90% of the distribution of the exam scores will be to the left of her score)

we get 93.5339641 = 94 when rounded to the nearest integer.

D = 95, normalcdf(-999999, 97, 82, 9) = .9522096696 = 95% when rounded to the nearest percent.

A+B+C+D = 2+89+94+95 = 280

15.) 11721

For parts A-D we need to plug into L1 all of the data then run 1 Var stats

A = $\mathbf{37.5}$ *or* $\mathbf{\frac{75}{2}}$, the mean of the data is symbolized as $\bar{x} = 37.5 \ or \ \frac{75}{2}$

B = $\mathbf{11.71}$ or $\mathbf{\frac{1171}{100}}$, the standard deviation is symbolized as $S_x = 11.70838191 = 11.71$ or $\frac{1171}{100}$ when rounded.

C = 31, the mode of the distribution is the most occurring score. Sadly, 1-Var stats doesn't calculate this but we can see that the value 31 is the most occurring (it occurs 3 times).

D = 37, the median is symbolized as Med = 37 which is the middle most number of the distribution.

100(A+B+C+D) = $\mathbf{100\left(\frac{75}{2} + \frac{1171}{100} + 31 + 37\right) = 100(117.21) = 11721}$