For all questions on this test, answer choice E is NOTA, indicating None Of The Above answers are correct.

(1) What is the maximum number of times a function may intersect its own horizontal asymptote?

(a) 0 (b) 1
(c) 2 (d) There is no maximum (e) NOTA

(2) A conic section $4x^2 + 160xy + 2021y^2 + 137x - 716y + 25 = 0$ is rotated by an angle θ to make an equation of the form $A'x^2 + C'y^2 + D'x + E'y + F' = 0$. What is $\frac{F'A'C'}{A'+C'}$?

(a) $\frac{1684}{81}$ (b) $-\frac{1684}{81}$

(c) $\frac{1684}{2025}$ (d) $-\frac{1684}{2025}$ (e) NOTA

(3) I have two points *A*: (4, 11) and *B*: (20, 21). Find the point *C* on the line segment $\overline{AB}$ connecting them for which $AC:CB = 1:3$.

(a) (12, 16) (b) (8, 13.5)
(c) (16, 27) (d) (24, 32) (e) NOTA

(4) A conic section is described by the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. This equation can be re-written using matrices, with the resulting equation being of the form $[x \quad y \quad 1]\boldsymbol{M}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$. If $\boldsymbol{M}$ is a 3×3 matrix, which of the following matrices is $\boldsymbol{M}$?

(a) $\begin{bmatrix} A & \frac{D}{2} & \frac{E}{2} \\ \frac{D}{2} & B & \frac{F}{2} \\ \frac{E}{2} & \frac{F}{2} & C \end{bmatrix}$ (b) $\begin{bmatrix} A & \frac{B}{2} & \frac{D}{2} \\ \frac{B}{2} & C & \frac{E}{2} \\ \frac{D}{2} & \frac{E}{2} & F \end{bmatrix}$

(c) $\begin{bmatrix} A & \frac{E}{2} & \frac{D}{2} \\ \frac{E}{2} & C & \frac{B}{2} \\ \frac{D}{2} & \frac{B}{2} & F \end{bmatrix}$ (d) $\begin{bmatrix} A & \frac{B}{2} & \frac{C}{2} \\ \frac{B}{2} & D & \frac{E}{2} \\ \frac{C}{2} & \frac{E}{2} & F \end{bmatrix}$ (e) NOTA

(5) How many solutions on $[0,2\pi]$ are there to the equation $\cos(x) + \cos(2x) + \cos(4x) = 0$?

(a) 4 (b) 6
(c) 8 (d) 10 (e) NOTA

(6) Find an expression for x, where defined, given that

$$x = \cos(\theta) + \cfrac{x}{\sin(\theta) + \cfrac{x}{\cos(\theta) + \cfrac{x}{\sin(\theta) + \cdots}}}$$

(a) $\cos(\theta) + \cot(\theta)$ (b) $\sin(\theta) + \cot(\theta)$
(c) $\cos(\theta) + \tan(\theta)$ (d) $\sin(\theta) + \tan(\theta)$ (e) NOTA

(7) If $0 < x < \frac{\pi}{4}$ and $\cos(x) + \sin(x) = \frac{5}{4}$, then what is $\cos(x) - \sin(x)$?

(a) $\frac{9}{16}$ (b) $\frac{\sqrt{7}}{4}$
(c) $\frac{25}{16}$ (d) $\frac{5\sqrt{7}}{16}$ (e) NOTA

(8) The two shortest sides of a right triangle are 2 and $\sqrt{2021}$. Find $\cos(\theta)$ where θ is the smallest angle in the triangle

(a) $\frac{\sqrt{2021}}{45}$ (b) $\frac{2}{45}$
(c) $\frac{\sqrt{2021}}{2021}$ (d) $\frac{1}{2}$ (e) NOTA

(9) Find the period of $f(x) = \cos(sin(x))$.

(a) 4π (b) 2π
(c) π (d) $\frac{\pi}{2}$ (e) NOTA

(10) $\cos(i) - i\sin(i)$ is the equivalent to

(a) $\frac{1}{e}$ (b) e
(c) $\frac{i}{e}$ (d) ie (e) NOTA

(11) Find the inverse of $\begin{bmatrix} \cos(\theta) & \cos(\theta) \\ -\sin(\theta) & \sin(\theta) \end{bmatrix}$ for $0 < \theta < \frac{\pi}{2}$.

(a) $\frac{1}{2}\begin{bmatrix} \sec(\theta) & -\csc(\theta) \\ \sec(\theta) & \csc(\theta) \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} \sec(\theta) & \sec(\theta) \\ -\csc(\theta) & \csc(\theta) \end{bmatrix}$

(c) $\frac{1}{2}\begin{bmatrix} \sec(\theta) & \csc(\theta) \\ -\sec(\theta) & \csc(\theta) \end{bmatrix}$ (d) $\frac{1}{2}\begin{bmatrix} \sec(\theta) & -\sec(\theta) \\ \csc(\theta) & \csc(\theta) \end{bmatrix}$ (e) NOTA

(12) Given the following system of equations:

$$\begin{gathered} 3x + 2y - z + 4w - 5p + 6q = 8 \\ x + 3y + 5w - 4p + 2q = 10 \\ 2x + 4y - 2z + 3p + 6q = 0 \\ 5y + z - w - 2p + 2q = -2 \\ 5x + 6y + 3z + 7w + 6q = 14 \\ x + 7y + 5z + w - p + 2q = 2 \end{gathered}$$

What is w?

(a) 0 (b) 1

(c) 2 (d) 4316 (e) NOTA

(13) If $M = \begin{bmatrix} \frac{1}{3} & 1 \\ -1 & \frac{1}{9} \end{bmatrix}$, find $X = M + M\left(M + M\left(M + M(\dots)\right)\right)$

(a) $\begin{bmatrix} \frac{1}{3} & 1 \\ -1 & \frac{1}{9} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{2}{3} & -1 \\ 1 & \frac{8}{9} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{3}{2} & -1 \\ 1 & \frac{9}{8} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & -1 \\ -1 & \frac{1}{8} \end{bmatrix}$ (e) NOTA

(14) If $i \equiv \sqrt{-1}$, simplify $\begin{bmatrix} \cos(\theta) & 3i \\ \frac{1}{3} i \cos(\theta) & 1 \end{bmatrix} \begin{bmatrix} 3 \\ \sin(\theta) \end{bmatrix}$.

(a) $\begin{bmatrix} 3e^{i\theta} \\ ie^{i\theta} \end{bmatrix}$ (b) $\begin{bmatrix} 3e^{-i\theta} \\ ie^{-i\theta} \end{bmatrix}$

(c) $\begin{bmatrix} 3e^{-i\theta} \\ ie^{i\theta} \end{bmatrix}$ (d) $\begin{bmatrix} 3e^{i\theta} \\ ie^{-i\theta} \end{bmatrix}$ (e) NOTA

(15) Simplify: $\begin{bmatrix}\cos(5°) & -\sin(5°)\\ \sin(5°) & \cos(5°)\end{bmatrix}^{2021}$

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}\cos(5°) & -\sin(5°)\\ \sin(5°) & \cos(5°)\end{bmatrix}$

(c) $\begin{bmatrix}\cos(10°) & -\sin(10°)\\ \sin(10°) & \cos(10°)\end{bmatrix}$ (d) $\begin{bmatrix}\cos(25°) & -\sin(25°)\\ \sin(25°) & \cos(25°)\end{bmatrix}$ (e) NOTA

(16) If vectors $\vec{u} = \langle 2,0,21\rangle$ and $\vec{v} = \langle x, 20, -21\rangle$ are perpendicular to one another, what is x?

(a) $\frac{441}{2}$ (b) 441

(c) $-\frac{441}{2}$ (d) -441 (e) NOTA

(17) What is the sine of the angle between the vectors $\vec{u} = \langle 1,2,2\rangle$ and $\vec{v} = \langle 2,6,9\rangle$?

(a) $\frac{32}{33}$ (b) $\frac{4\sqrt{2}}{33}$

(c) $\frac{\sqrt{65}}{33}$ (d) $\frac{1}{33}$ (e) NOTA

(18) Find the volume of the parallelepiped with edges corresponding to the vectors $\langle 1,2,3\rangle$, $\langle 2,1,-1\rangle$, and $\langle 2,5,2\rangle$.

(a) 19 (b) 16

(c) 15 (d) 12 (e) NOTA

(19) Find the center of the conic described by the equation $x^2 + 3y^2 - 4x + 6y + 18 = 0$.

(a) (2, 1) (b) (-2, 1)

(c) (-2, -1) (d) (2, -1) (e) NOTA

(20) Which of the following is NOT a degenerate conic?

(a) $x^2 + y^2 + 4x + 4 = 0$ (b) $x^2 + 2xy + y^2 + 2x + 2y + 1 = 0$
(c) $x^2 + 2xy + y^2 + 2x + 2y = 0$ (d) $xy - 2x + y - 2 = 0$
(e) NOTA

(21) Which is true about the eccentricity e of the non-degenerate conic $3x^2 - 6xy + 3y^2 + 3x - 4y + 1 = 0$?

(a) $e = 0$ (b) $0 < e < 1$
(c) $e = 1$ (d) $e > 1$ (e) NOTA

(22) What is the shortest distance between the line $3x - y + 2021 = 0$ and the line $3x - y - 2021 = 0$?

(a) 0 (b) $\frac{2021\sqrt{10}}{10}$
(c) $\frac{2021\sqrt{10}}{5}$ (d) $\frac{4042\sqrt{10}}{5}$ (e) NOTA

(23) What is the area contained within the parametric curve $\begin{cases} x = 4\sin(t) + 2021 \\ y = 5\cos(t) - 2021 \end{cases}$?

(a) 20π (b) 8084π
(c) 2021π (d) 4084441π (e) NOTA

(24) The slant asymptote of $y = 2021x + 1 + \frac{1}{x+1}$ is perpendicular to the slant asymptote of $y = \frac{kx^2+x+1}{2021x+1}$. What is a possible value of k?

(a) $\frac{1}{2021}$ (b) $-\frac{1}{2021}$
(c) -2021 (d) -1 (e) NOTA

(25) Find the maximum possible value of $x^2 + y^2 + z^2$ given that $x^4 + y^4 + z^4 = 2021$.

(a) $\sqrt{2021}$ (b) $\sqrt{6063}$
(c) 2021 (d) 6063 (e) NOTA

(26) What is the area of a triangle with side lengths a, b, and c, if a is the number of questions on #1 to #25 of this test with correct answer A, b is the number of questions on #1 to #25 of this test with correct answer B, and c is the number of questions on #1 to #25 of this test with correct answer C?

(a) 12 (b) $2\sqrt{6}$

(c) 36 (d) $6\sqrt{6}$ (e) NOTA

(27) How many arrangements are there of the letters in the correct answers for the first ten questions of this test?

(a) 6300 (b) 12600

(c) 25200 (d) 50400 (e) NOTA

(28) Find the sum of the solutions to $\tan(Nx) = 1$ for $0 < x < 2\pi$, if N is the number of questions with correct answer D among the questions #1 to #25 of this test.

(a) 30π (b) 7π

(c) 69π (d) 39π (e) NOTA

(29) Let M be numerical value of the largest correct answer that is also an integer among problems #1 to #25 on this test. Find the coefficient of the constant term of $\left(x + \frac{1}{2x}\right)^{2M}$.

(a) $\binom{16}{8}\frac{1}{2^8}$ (b) $\binom{30}{15}\frac{1}{2^{15}}$

(c) $\binom{38}{19}\frac{1}{2^{19}}$ (d) $\binom{24}{12}\frac{1}{2^{12}}$ (e) NOTA

(30) The answer to this question is the modal answer among correct answers for questions #1 to #25 of this test.

(a) A (b) B

(c) C (d) D (e) NOTA