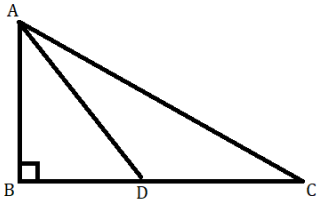
NOTA: none of the above answers is correct

1) ΔABC has $m\angle ACB = 50^o$ and the $m\angle BCA$ is 20 less than double $m\angle BAC$. What is the largest angle measurement of ΔABC?

a) 20
b) 85
c) 95
d) 110
e) NOTA

2) Right triangle ABC has a right angle at B and an area of 60 square units. D lies on $\overline{BC}$ such that the ratio of $\overline{BD}:\overline{DC} = 2:3$. If the length of $\overline{AB}$ is 8, then what is the perimeter of ΔADC?



a) $16+\sqrt{145}$
b) $19+\sqrt{145}$
c) 33
d) 36
e) NOTA

3) Line segment $\overline{AB}$ contains points C, D, and E. If C is the midpoint of $\overline{AB}$, D is the midpoint of $\overline{AC}$, and E is the midpoint of $\overline{AD}$. Given that the length of $\overline{AB} = 40$, then what is the length of $\overline{CE}$?

a) 5
b) 10
c) 15
d) 20
e) NOTA

4) An isosceles triangle has vertex angle x and base angle y. Which of the following statements could <u>never</u> be true?

a) complement of x = supplement of y
b) supplement of x = complement of y
c) complement of x = complement of y
d) supplement of x = supplement of y
e) NOTA

5) "If you teach a man to fish, then you feed him for a lifetime." Given that this proverb is true, using logic, which other statement must also be true?

a) If you give a man a fish then you feed him for a day
b) If you feed a man for a lifetime, then you taught him to fish.
c) If you do not teach a man to fish, then you do not feed him for a lifetime.
d) If you do not feed a man for a lifetime, then you did not teach him to fish.
e) NOTA

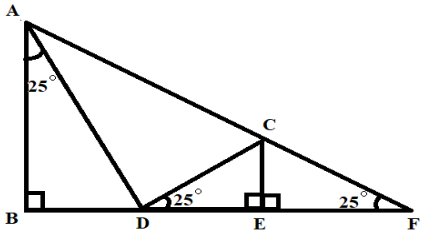
6) Line A has the equation $2x - 5y = 10$ and travels through the coordinate $(0, c)$. Given that line A $\perp$ line B, and line B travels through the coordinate $(c, 0)$, then what is the equation of line B?

a) $5x + 2y = 10$
b) $5x + 2y = -10$
c) $2x + 5y = 10$
d) $2x + 5y = -10$
e) NOTA

7) An isosceles triangle has two legs of 6 units. What is a possible length of the third side such that the triangle is also obtuse?

a) 6
b) 8
c) 9
d) 12
e) NOTA

8) Given the diagram, what is the $m\angle DCA$?



a) 25^o b) 30^o c) 50^o d) 65^o e) NOTA

9) Two people leave a grocery store parking lot to take groceries to their respective houses. The driving distances to each home is 20 miles from the store. The first drives north the entire way to get home. The other began north, but then turned west after 5 miles of driving, and continues west to reach their home. What is the shortest direct distance between the two homes on a map?

a) $15\sqrt{2}$ miles
b) $20\sqrt{2}$ miles
c) 25 miles
d) 35 miles
e) NOTA

10) A plane can be defined by the following:
I. Two distinct coordinates on the plane
II. Two parallel lines on the plane
III. Two perpendicular lines on the plane

a) I only
b) I and III only
c) II and III only
d) I, II and III
e) NOTA

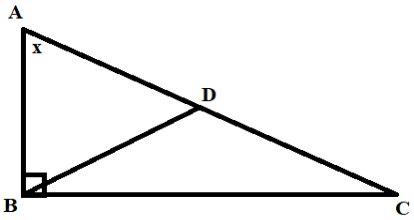
11) The perimeter of a right triangle with integral sides is 24. What is the absolute value of the difference between the shortest and longest altitudes of the triangle?

a) $6/5$
b) $12/5$
c) 16/5
d) 24/5
e) NOTA

12) A child places two regular octagons next to each other such that they share a common vertex. What is the third regular shape the child could add to the vertex in order to fill in the open space at that common vertex?

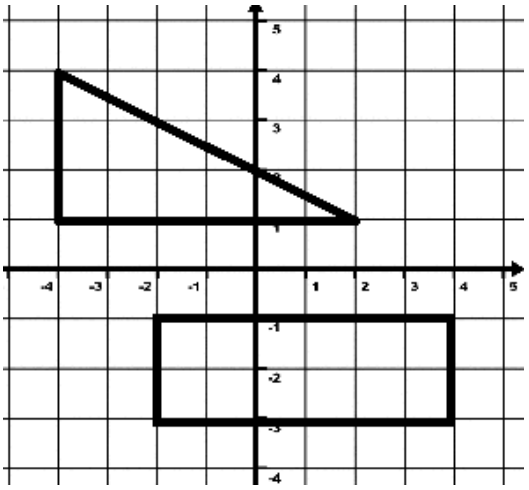
a) Triangle
b) Quadrilateral
c) Hexagon
d) Octagon
e) NOTA

13) Given that $\tan(x) = \frac{4}{3}$ and $\overline{BD}$ is the median to the hypotenuse of ΔABC, then what is the ratio of $\overline{BD}$: $\overline{BC}$?



a) 16:25
b) 9:16
c) 5:6
d) 5:8
e) NOTA

14) What is the distance between the center of masses of the triangle and rectangle?

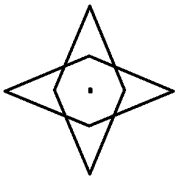


a) 2 b) $2\sqrt{3}$ c) $3\sqrt{2}$ d) 5 e) NOTA

15) A person cuts a 9-unit long thread into three integral pieces to form one large triangle by connecting the endpoints of the pieces. How many unique triangles could have been formed using this process?

a) 1
b) 2
c) 3
d) 4
e) NOTA

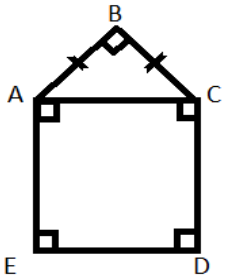
16) Two congruent rhombii are place atop one another at a shared center, but one is rotated 90 degrees about its center as shown in the diagram below. Given that the diagonals of each were 10 and 24 units, then what is the length of one side of the equilateral octagon formed inside the rhombii?



a) 130/12
b) 65/12
c) 130/17
d) 65/17
e) NOTA

17) A pentagon ABCDE is formed by placing an isosceles right triangle ABC on top of square ACDE as shown. Given that the longest diagonal of the pentagon is $\sqrt{10}$, then what is the perimeter of the pentagon?

a) $6 + 2\sqrt{2}$
b) $3 + 2\sqrt{2}$
c) $3\sqrt{5} + \sqrt{10}$
d) $3\sqrt{5} + 2\sqrt{10}$
e) NOTA



18) The exterior angle of a regular polygon is x degrees. If another regular polygon has x diagonals, then which is <u>not</u> a possible value of x?

a) 2
b) 5
c) 9
d) 15
e) NOTA

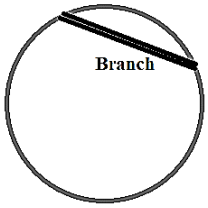
19) A 1-foot long wire is folded into a square. How many inches is the radius of the incircle of the square?

a) 0.5 b) 1.5 c) 3 d) 6 e) NOTA

20) A person parasailing at a height of 500 feet above the ground. At an angle of depression of 30^o they see the very top of a 100-foot tall lighthouse. What is the distance between the parasailer and the base of the lighthouse?

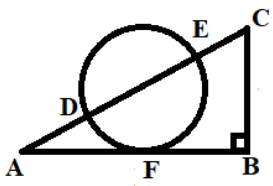
a) $100\sqrt{73}$
b) $200\sqrt{7}$
c) $400\sqrt{3}$
d) 800
e) NOTA

21) A 14-inch diameter pipe has a 10-inch branch lodged into it as shown. What is the shortest distance the branch is from the center of the pipe?



a) $6\sqrt{19}$
b) $3\sqrt{19}$
c) $4\sqrt{6}$
d) $2\sqrt{6}$
e) NOTA

22) The right triangle ΔABC intersects the circle with secant line $\overline{DE}$ and tangent $\overline{AB}$ as shown. Given that $\widehat{DE}$ above the triangle is 200^o, $\overline{CB} = 20$, $\overline{AB} = 20\sqrt{3}$, then what is the degree measurement of minor arc $\widehat{EF}$?

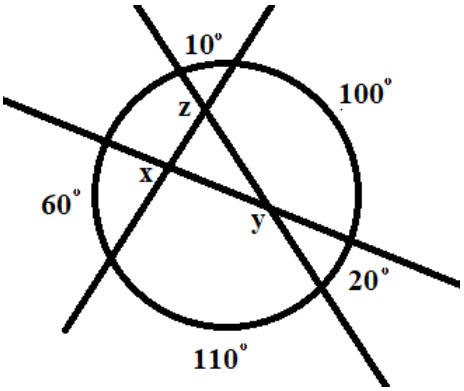


a) 50
b) 70
c) 100
d) 110
e) NOTA

23) What is the closest distance the circle with the equation $(x-3)^2 + (y+4)^2 = 4$ is to the origin?

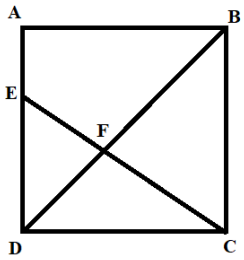
a) 1 b) 3 c) 5 d) 7 e) NOTA

24) What is the sum of the measures of x, y, and z for the diagram below?



a) 180^o
b) 320^o
c) 340^o
d) 360^o
e) NOTA

25) Square ABCD has point E on $\overline{AD}$ such that $\overline{AE}:\overline{ED} = 1:2$. Point F is the intersection of segment $\overline{CE}$ and diagonal $\overline{BD}$. What is the ratio of the area of $\Delta EFD:\Delta BFC$?



a) 4/9
b) 2/9
c) 2/3
d) 1/3
e) NOTA

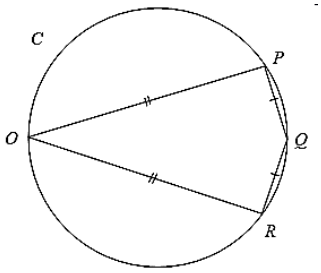
26) A person draws four distinct lines that each intersect a circle and share a single point of concurrency. Which of the following is **not** a possible number of sectors formed within the circle based upon the description?

a) 3
b) 4
c) 5
d) 8
e) NOTA

27) An isosceles trapezoid with perimeter of 5 has three sides that each have a length of 1. What is the height of the trapezoid?

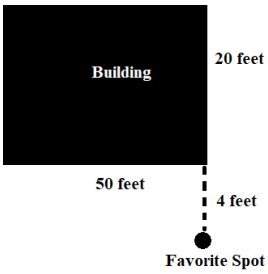
a) $\frac{1}{2}\sqrt{5}$
b) $\frac{1}{2}\sqrt{3}$
c) $\frac{1}{2}\sqrt{2}$
d) $\frac{1}{2}$
e) NOTA

28) Circle C is the circumcircle of kite $PQRO$ such that diagonal $\overline{QO}$ is also the diameter of the circle. If $\overline{QO} = 17$ and $\overline{PR} = 8$, then what is the perimeter of the kite?



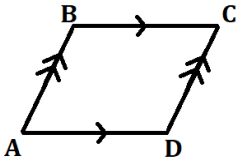
a) $10\sqrt{17}$
b) $5\sqrt{17}$
c) $15\sqrt{2}$
d) $15\sqrt{3}$
e) NOTA

29) A toddler is playing outside of their home. However, the toddler always stays within 8 feet of their favorite spot as shown. How much total space does the toddler have to roam?



a) $\frac{160\pi+8\sqrt{3}}{3}$
b) $\frac{160\pi+24\sqrt{3}}{3}$
c) $48\pi + 8\sqrt{3}$
d) $48\pi + 8\sqrt{2}$
e) NOTA

30) What is the value of $x + y$ given that $m\angle BAD = 5y - 15$ and $m\angle ABC = 5x + 10$?



a) 37
b) 35
c) 25
d) Insufficient Info
e) NOTA

February Dunbar Statewide 2021 – Geometry Answers

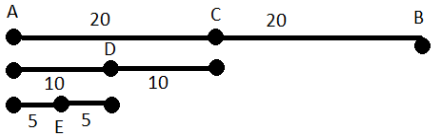
1. C	**2. D**	**3. C**	**4. A**	**5. D**	**6. B**	**7. C**	**8. C**	**9. A**	**10. C**
11. C	**12. B**	**13. D**	**14. D**	**15. C**	**16. D**	**17. A**	**18. D**	**19. B**	**20. A**
21. D	**22. D**	**23. B**	**24. C**	**25. A**	**26. E**	**27. B**	**28. A**	**29. B**	**30. A**

February Dunbar Statewide 2021 – Geometry Solutions

1) **C.** $\angle C = 50 = 2\angle A - 20 \rightarrow \angle A = 35$. Therefore, the three angles are $35^o, 50^o, 95^o$.

2) **D.** Using the area of the triangle, $A = \frac{1}{2}bh \rightarrow 60 = \frac{1}{2}(b)(8) \rightarrow b = 15$. Since $\overline{BC}$ is split into a 2:3 ratio, we can determine that $\overline{BD} = 6$ and $\overline{DC} = 9$. Therefore, using Pythagorean Theorem (or 6-8-10, 8-15-17 triples), we can determine the lengths of $\overline{AD}$ is 10 and $\overline{AC}$ is 17. The perimeter of ΔADC is 10+17+9 = 36.

3) **C.** Drawing the segment and cutting it up into the midpoints you can find that the length is going to be 15 as shown in the diagram below.



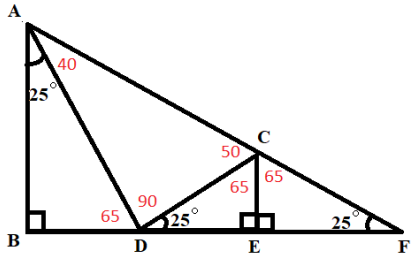
4) **A.** Isosceles triangle would have angles $x + y + y = 180$. Since y will appear twice, you know that it must be under 90^o. Therefore, the supplement of y must be greater than 90^o. The complement of x will always be less than 90^o. So the statement that must be false is that the complement of x = supplement of y. Choice B is satisfied by a 30-30-120 triangle, choice C and D are satisfied by a 60-60-60 triangle.

5) **D.** The statement and contrapositive always match in truth value. The contrapositive would be: "If you do not teach a man for a lifetime, then you did not teach him to fish."

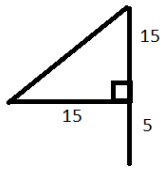
6) **B.** Slope of line A is $\frac{2}{5}$ and y-intercept is $(0, -2)$. Using definition of perpendicular lines, line B must have slope of $-\frac{5}{2}$ and it travels through $(-2,0)$ as described in the problem statement. Then use point slope form, and a bit of algebra: $y - 0 = -\frac{5}{2}(x - (-2)) \rightarrow y = -\frac{5}{2}x - 5 \rightarrow 2y = -5x - 10 \rightarrow 5x + 2y = -10$.

7) **C.** $c^2 > a^2 + b^2$, in order for an obtuse triangle to also be isosceles, the legs would have to be the shorter sides. Therefore $c^2 > 36 + 36 \rightarrow c^2 > 72$. Any value greater than $\sqrt{72}$ meet the criteria. However, to make a triangle $a + b > c$, therefore $c < 12$. The only listed number meeting this is 9.

8) **C.** The sum of the angles of triangles are 180^o and using that allows you to find the angles as shown.



9) **A.** The paths form a 45-45-90 triangle of side lengths 15. The distance between the two homes would be $15\sqrt{2}$.



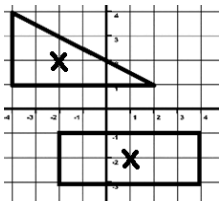
10) **C.** A plane can be defined by 3 non-collinear points. Two parallel lines and two perpendicular lines on the plane meet that criteria. Only two distinct coordinates would be insufficient.

11) **C.** Using guess-and-check of Pythagorean Triples, the triangle has side lengths of 6, 8 and 10. The longest altitude is easy, 8. The smallest altitude can be found by using the area of the triangle with different side lengths of $\frac{BH}{2}$. $\frac{6\cdot 8}{2} = \frac{10\cdot Alt}{2} \rightarrow$ Smallest altitude is $\frac{48}{10} = \frac{24}{5}$. Finding the difference between $8 - \frac{24}{5} = \frac{16}{5}$

12) **B.** The interior angles for a regular octagon are 135^o. The combined of the two are 270^o. Completing the circle of 360^o, the 3rd shape needs an interior angle of 90^o. Therefore it must be a regular quadrilateral.

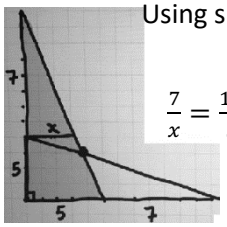
13) **D.** Since $\tan(x) = \frac{4}{3}$, you can assume ΔABC is a 3:4:5 right triangle. The median of $\overline{BD}$ is half of the hypotenuse, 2.5. The ratio of $\overline{BD}: \overline{BC} = 2.5: 4$, which can be written as $5: 8$.

14) **D.** The two centroids (center of masses) are at $(-2,2)$ and $(1,-2)$ as shown below. These can be found visually or averaging the vertices. The distance between them for a 3-4-5 right triangle, so the distance is 5.

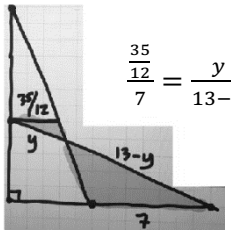


15) **C.** $a + b + c = 9$, but also $a + b > c$. The only possible side lengths are 1-4-4, 2-3-4, and 3-3-3.

16) **D.** Since rhombii have perpendicular bisectors you form a right triangle of 5-12-13 as shown in the diagrams. Using similar triangles solve the proportions below:



$\frac{7}{x} = \frac{12}{5} \rightarrow x = \frac{35}{12}$



$\frac{\frac{35}{12}}{7} = \frac{y}{13-y} \rightarrow \frac{5}{12} = \frac{y}{13-y} \rightarrow 65 - 5y = 12y \rightarrow 65 = 17y \rightarrow y = \frac{65}{17}$

17) **A.** The pentagon has five diagonals. The longest diagonals would be $\overline{BD} = \overline{BE} = \sqrt{10}$. Lets define F as the midpoint of side $\overline{DE}$. If we state that $\overline{FE} = x$, then we can also state that $\overline{FB} = 2x + x = 3x$. Therefore, the length of diagonal $\overline{BE} = \sqrt{9x^2 + x^2} = \sqrt{10x^2} = \sqrt{10}$. So, $x = 1$ and the perimeter of the pentagon would be $2 + 2 + 2 + \sqrt{2} + \sqrt{2} = 6 + 2\sqrt{2}$.

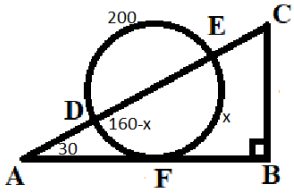
18) **D.** Exterior angles are $\frac{360}{n}$ for a regular polygon. Meanwhile the diagonals of a polygon are $\frac{n(n-3)}{2}$. You need to find integral values for both. The integral exterior values of angles would be as follows: $1^o, 2^o, 3^o, 4^o, 5^o, 6^o, 8^o, 9^o, 10^o, 12^o, 15^o, 18^o, 20^o, 24^o, 30^o, 36^o, 40^o, 45^o, 60^o, 72^o, 90^o, 120^o$ Then list out the possible diagonals as follows (which won't take long because you can stop when you get above 120): 0,2,5,9,14,20,27,35,44,54,65,77,90,104,119. The only matching numbers were 2, 5, 9, 20, 90.

19) **B.** The square would have a side length of 3 inches. The incircle radius would be half a side length (1.5").

20) **A.** Using a 30-60-90 triangle the horizontal distance is $400\sqrt{3}$. The vertical distance is 500. Scaling the triangles down by a factor of 100, find distance $\sqrt{4\sqrt{3}^2 + 5^2} = \sqrt{48 + 25} = \sqrt{73}$. Scale it back up $100\sqrt{73}$.

21) **D.** Using the perpendicular bisector, it forms a right triangle $x - 5 - 7$. Using the Pythagorean Theorem, the shortest distance, x, is $\sqrt{7^2 - 5^2} = \sqrt{49 - 25} = \sqrt{24} = 2\sqrt{6}$.

22) **D.** The lengths of the triangles is such that you know that it is a 30-60-90 triangle such that $\angle A = 30^o$. Using the secant-tangent relationship, you can state $\angle = \frac{Arc_1 - Arc_2}{2}$.
$30 = \frac{x-(160-x)}{2} \rightarrow 60 = 2x - 160 \rightarrow x = 120$

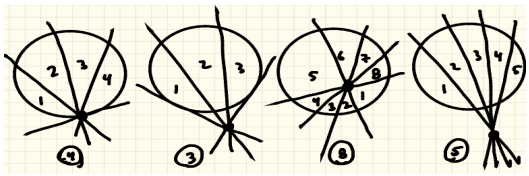


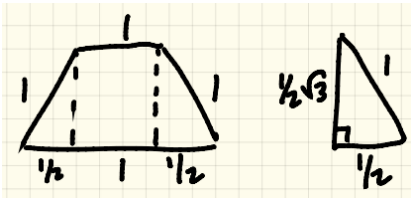
23) **B.** Center is $(3, -4)$ with a radius of 2. The distance to the origin from the edge of the circle is $5 - 2 = 3$

24) **C.** The chord-chord relationship is $\frac{Arc_1 + Arc_2}{2}$ can be used throughout the diagram. The equations are as follows: $x = \frac{100+60}{2} = 80$, $y = \frac{110+60+100+10}{2} = 140$, $z = \frac{60+60+100+20}{2} = 120$. Sum of $x + y + z = 340$

25) **A.** From the ratio provided you can determine that the ratio of the side lengths of the triangles is 2:3. Therefore the ratio of the areas would be $(2:3)^2$ or 4:9 because they are similar to one another.

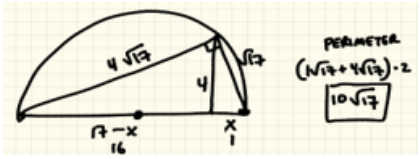
26) **E.** All of the options work as shown in the diagrams below





27) **B.** The isosceles trapezoid will have side lengths of 1-1-1-2. It forms a 30-60-90 triangle as shown above. Therefore, the height would then be $\frac{1}{2}\sqrt{3}$.

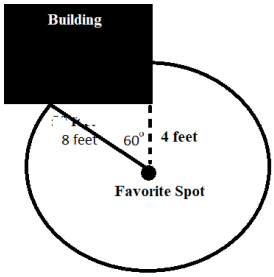
28) **A.** Split the circle into two semi-circles along diameter OQ. By definition triangle QPQ must be a right triangle, with hypotenuse 17 and altitude of 4 (half of PR). Using similarity and formulas associated with altitudes of right triangles, you can set up the equation below:



$4^2 = x(17 - x) \rightarrow 16 = 17x - x^2 \rightarrow x^2 - 17x + 16 = 0 \rightarrow x = 16$ or 1

Then use the Pythagorean Theorem to find the side lengths of the kite, $4^2 + 16^2 = (s_1)^2$ and $1^2 + 4^2 = (s_2)^2$. The $s_1 = 4\sqrt{17}$ and $s_2 = 1\sqrt{17}$. Therefore, perimeter of the kite is $2(s_1 + s_2) = 2\left(4\sqrt{17} + 1\sqrt{17}\right) = 10\sqrt{17}$

29) **B.** The roaming area forms $\frac{5}{6}$ of a circle and a triangle as shown



$\frac{5}{6}\pi r^2 + \frac{1}{2}BH = \frac{5}{6}\pi(8)^2 + \frac{1}{2}(4)\left(4\sqrt{3}\right) = \frac{160}{3}\pi + 8\sqrt{3} = \frac{160\pi+24\sqrt{3}}{3}$

30) **A.** $\angle ABC + \angle BAC = 180^o$
$5y - 15 + 5x + 10 = 180$
$5x + 5y = 195$
$x + y = 37$