1. Answer the following regarding the sets of numbers, A and B listed below:

$$A = \left\{\frac{8}{1}, \frac{7}{2}, \frac{6}{3}, \frac{5}{4}, \frac{4}{5}, \frac{3}{6}, \frac{2}{7}, \frac{1}{8}\right\} \text{ and } B = \{\sqrt{0}, \sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}\}$$

   A = The product of all rational numbers in set A
   B = The product of all real numbers in set B
   C = The product of all distinct natural numbers that are in either set A or set B
   D = The number of distinct integers that are in both set A and set B
   Answer = $A + 10B + 100C + 1000D$

2. Answer the following questions regarding the factors of the values below:
   A = The number of distinct positive integral factors of the number 144
   B = The sum of all distinct positive integral factors of 120
   C = The product of all distinct positive factors of 16
   D = The number of 2-digit prime numbers that are 2 more than a perfect square or perfect cube

   Answer = $\frac{100A+10B}{D} - C$

3. A coordinate plane has three lines. Line A has the equation $2x + y = 4$, Line B has the equation $x - 3y = 6$ and Line C has the equation $y - 2 = 3(x - 1)$.

   A = The arithmetic mean of the y-intercept values of lines A, B, and C.
   B = The arithmetic mean of the slopes of lines A, B, and C.
   C = The abscissa (x-value) of the point of intersection of Line A and Line C.
   D = The ordinate (y-value) of the point of intersection of Line B and Line C.

   Answer = $3A + 9B + 5C + 8D$

4. Find the sum of the <u>absolute values</u> of the solutions of the quadratic equations below.

   A = $|a_1| + |a_2|$ given that the solutions to $x^2 - 5x + 6 = 0$ are $x = a_1$ and $x = a_2$
   B = $|b_1| + |b_2|$ given that the solutions to $8x^2 - 2x - 3 = 0$ are $x = b_1$ and $x = b_2$
   C = $|c_1| + |c_2|$ given that the solutions to $(x - 4)^2 - 4 = 0$ are $x = c_1$ and $x = c_2$
   D = $|d_1| + |d_2|$ given that the solutions to $4\left(x - \frac{1}{2}\right)^2 - 4 = 0$ are $x = d_1$ and $x = d_2$

   Answer = $A + B \cdot C - D$

5. Answer the following questions regarding the key features of the absolute value function below:

$$y = 2|x - 1| - 3$$

A = The minimum y-value of the graph of the function.
B = The sum of all possible values of B such that the graph travels through the coordinate $(B, 1)$
C = The y-value of the y-intercept of the graph of the function.
D = The area of the triangle bounded by the intersection of the graph and the x-axis

Answer = $A + B + C + 2D$

6. Simplify the following expressions and determine the missing values.

A = The value of A such that $((x^2)^3)^A = x^{234}$
B = The value of B such that $\sqrt[4]{\sqrt[3]{y^B}} = y^2$
C = The value of C such that $\frac{z^4(z^3)^2}{z^C} = z^{-1}$
D = The value of D such that $D^D = 2^8$

Answer = $(A + B) - (C + D)$

7. Tim, Tom, Tam, and Tum competed in an aquathlon consisting of swimming 1 kilometer, and running 2.5 kilometers. Tim runs three times faster than he swims. Tom runs and swims at the same speed. Tam swims twice as fast as she runs. Tum swims at two-fifths of the speed that she runs. All four maintain constant speeds without breaks during the aquathlon.

A = If Tim completed the swimming portion of the aquathlon in 18 minutes, then how many minutes did it take Tim to complete the running portion of the aquathlon?
B = If Tom completed the entire aquathlon in 42 minutes, then how many minutes did it take Tom to complete the swimming portion of the aquathlon?
C = If Tam completed the swimming portion of the aquathlon in 10 minutes, then how many minutes did it take Tam to complete the running portion of the aquathlon?
D = If Tum completed the running portion of the aquathlon in 30 minutes, then how many minutes did it take Tum to complete the swimming portion of the aquathlon?

Answer = $A + B + C + D$

8. The solution to the system of equations $Ax + By = 14$ and $2x - 3y = C$ is $(4,2)$. The solution to the system of equations $Ax + By = 14$ and $2x + 7y = D$ is $(5, -1)$.
   A = The value of A in the equation $Ax + By = 14$
   B = The value of B in the equation $Ax + By = 14$
   C = The value of the constant C
   D = The value of the constant D

   Answer = $A \cdot B \cdot C \cdot D$

9. Answer the following questions regarding the key features of the parabola with the equation:
   $$y = x^2 - 10x + 9$$
   A = The ordinate value the y-intercept of the parabola
   B = The horizontal distance between the x-intercepts of the parabola
   C = The abscissa value of the vertex of the parabola
   D = The minimum ordinate value of the parabola

   Answer = $A + B + C + D$

10. Given that $a@b = a^b - b^a$

    A = The value of $1@2$
    B = The value of $2@1$
    C = The value of $3@\text{-}1$
    D = The value of $1@(2@3)$

    Answer = $A + 2B + 3C + 4D$

11. Calculate the following ages from the riddles outlined below:
    - Abraham, Brandon, Carrie and Dorothy were all born on January 1 of different years.
    - In ten years, Abraham will be exactly double the age he was one year ago.
    - Right now, Brandon is as old as the sum of the ages of his first five birthdays combined.
    - Three years ago, Carrie was two-thirds of the age that she will be two years from now.
    - One less than doubling Dorothy's age is the same as tripling her age 5 years ago.

    A = Abraham's current age in years
    B = Brandon's current age in years
    C = Carrie's current age in years
    D = Dorothy's current age in years

    Answer = $A + B + C + D$

12. A person wants to buy an item at a store that is currently marked as \$20. (Assume no sales tax)
    A = The number of dollars the item costs with a coupon for 20% off
    B = The old price of the item in dollars if the marked price, \$20 is already 20% off of the original price.
    C = The number of ways to pay the exact \$20 price only \$1-bills, \$5-bills, \$10-bills, or \$20-bills.
    D = The number of ways to receive exact change if you bought the \$20 item with a \$50 bill and got change with only \$1-bills, \$5-bills, \$10-bills, or \$20-bills.

    Answer $= (B - A) \cdot (D - C)$

13. Find the number of integral solutions to the equations and inequalities below:

    A = The number of integral solutions to $|x + 2| = 10$
    B = The number of integral solutions to $8 - 2|x| = 10$
    C = The number of integral solutions to $|x| \leq 4$
    D = The number of integral solutions to $|2x - 8| < 4$

    Answer = $(B + C + D)^A$

14. Simplify each of the following expressions:

    A = $2(3 - 4)^5$
    B = $\frac{4+3^2-1}{2^3-4}$
    C = $\sqrt{10^2 - 10(4) + 4}$
    D = $\frac{4^2}{2^{-2}}$

    Answer = $A + B + C + D$

15. For each of the statements, write the value corresponding to the truth value as outlined below:
    - If the statement is <u>always</u> true, then the value is 2.
    - If the statement is <u>sometimes</u> true, then the value is 1.
    - If the statement is <u>never</u> true, then the value is 0.

    A = A real number ($\mathbb{R}$) plus an integer ($\mathbb{Z}$) is a rational number ($\mathbb{Q}$)
    B = An irrational number ($\mathbb{P}$) times a real number ($\mathbb{R}$) is a rational number ($\mathbb{Q}$)
    C = An integer ($\mathbb{Z}$) multiplied by a natural number ($\mathbb{N}$) is an integer ($\mathbb{Z}$)
    D = An irrational number ($\mathbb{P}$) raised to a power that is a natural number ($\mathbb{N}$) is an integer ($\mathbb{Z}$)

Answer = $1000A + 100B + 10C + D$

Team Answers:

Part	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14	#15
A	1	15	1/3	5	-3	39	15	3	9	-1	12	16	2	-2	1
B	0	360	4/9	5/4	2	24	12	1	8	1	15	25	0	3	1
C	48	1024	1	8	-1	11	50	2	5	4/3	13	10	9	8	2
D	1	3	-17/8	2	9/2	4	30	3	-16	0	14	20	3	64	1
Final	5801	676	-7	13	7	48	107	18	6	5	54	90	144	73	1121

#	Answer	Worked Out Solution
1	A=1 B=0 C=48 D=1 <u>Answer</u> 5801	a) All values in set A are rational values because a rational value is one that can be represented as a fraction. The numerators and denominators will all cancel one another out. Therefore, the product is 1. b) All values in Set B are real numbers. Since one them is 0, the product will be 0. c) The combined set of natural numbers is $A \cup B$: $\{1,2,3,8\}$ and the product is 48. d) The distinct integers that are in both sets is $A \cap B$: $\{2\}$ therefore there is only 1 value. Answer = $A + 10B + 100C + 1000D = 1 + 0 + 4800 + 1000 = 5801$
2	A=15 B=360 C=1024 D=3 <u>Answer</u> 676	a) $144 = 2^4 \cdot 3^2$. The number of factors is the product of 1 more than the degrees of the prime factorization, $(4+1)(2+1) = 5 \cdot 3 = 15$. b) $120 = 2^3 \cdot 3^1 \cdot 5^1$. The sum of the factors is $(2^3 + 2^2 + 2^1 + 2^0)(3^1 + 3^0)(5^1 + 5^0)$; $15 \cdot 4 \cdot 6 = 60 \cdot 6 = 360$ c) The factors of 16 are $1 \times 16, 2 \times 8, 4 \times 4$. The distinct positive factors are 1,2,4,8,16. This is equivalent to $2^{0+1+2+3+4} = 2^{10} = 1024$ d) The perfect squares are 1,4,9,16,25,36,49,64,81 and the only prime numbers that are two more than those values are 11, 83. The perfect cubes are 1,8,27,64 and the only prime number that is two more than those values is 29. Therefore, there are 3 total values. Answer = $\frac{100A+10B}{D} - D = \frac{1500+3600}{3} - 1024 = 1700 - 1024 = 676$
3	A=1/3 B=4/9 C=1 D=-17/8 Answer -7	Line A: $2x + y = 4 \rightarrow y = -2x + 4$ Line B: $x - 3y = 6 \rightarrow y = \frac{1}{3}x - 2$ Line C: $y - 2 = 3(x-1) \rightarrow y - 2 = 3x - 3 \rightarrow y = 3x - 1$ a) The arithmetic mean of the y-intercept values is $\frac{4-2-1}{3} = \frac{1}{3}$ b) The arithmetic mean of the slopes is $\frac{-2+\frac{1}{3}+3}{3} = \frac{\frac{4}{3}}{3} = \frac{4}{9}$ c) Rearrange line A into slope-intercept form: $2x + y = 4 \rightarrow y = -2x + 4$. Then substitute the equation in for line C, $(-2x + 4) - 2 = 3(x-1)$. After simplification the equation becomes $-2x + 2 = 3x - 3 \rightarrow -5x = -5 \rightarrow x = 1$. The abscissa, x-value, of the intersection is 1. d) Rearrange line B solving for x, $x = 6 + 3y$. Substitute the expression into the equation of line C, $y - 2 = 3(6 + 3y - 1)$. After simplification and distribution the equation becomes $y - 2 = 3(5 + 3y) \rightarrow y - 2 = 15 + 9y \rightarrow -17 = 8y \rightarrow y = -\frac{17}{8}$. Therefore the ordinate, y-value, of the intersection is $-\frac{17}{8}$.

		Answer = $3A + 9B + 5C + 8D = 1 + 4 + 5 - 17 = 10 - 17 = -7$
4	A=5 B=5/4 C=8 D=2 <u>Answer</u> 13	a) $x^2 - 5x + 6 = 0 \to (x-2)(x-3) = 0 \to x = 2,3 \;\; \|2\| + \|3\| = 5$ b) $8x^2 - 2x - 3 = 0 \to (2x+1)(4x-3) = 0 \to x = -\frac{1}{2}, \frac{3}{4} \;\; \left\|-\frac{1}{2}\right\| + \left\|\frac{3}{4}\right\| = \frac{5}{4}$ c) $(x-4)^2 - 4 = 0 \to x^2 - 8x + 16 - 4 = 0 \to x^2 - 8x + 12 = 0 \to (x-2)(x-6) = 0$ $x = 2,6 \;\; \|2\| + \|6\| = 8$ d) $4\left(x - \frac{1}{2}\right)^2 - 4 = 0 \to 4\left(x^2 - x + \frac{1}{4}\right) - 4 = 0 \to 4x^2 - 4x + 1 - 4 = 0 \to 4x^2 - 4x - 3 = 0,$ $(2x-3)(2x+1) = 0 \to x = \frac{3}{2}, -\frac{1}{2} \;\; \left\|\frac{3}{2}\right\| + \left\|-\frac{1}{2}\right\| = 2$ Answer = $A + B \cdot C - D = 5 + \frac{5}{4} \cdot 8 - 2 = 5 + 10 - 2 = 13$
5	A=-3 B=2 C=-1 D=9/2 <u>Answer</u> 7	a) Minimum y-value is the vertex y-value, -3. b) Plug in 1 for y and solve, $1 = 2\|x-1\| - 3 \to 2 = \|x-1\| \to x = 3, -1$; sum is 2 c) Plug in $x = 0$ and solve for y, $2\|-1\| - 3 = 2 - 3 = -1$. d) Plug in $y = 0$ and solve for the x-intercepts; $\frac{3}{2} = \|x-1\| \to x = \frac{5}{2}$ and $-\frac{1}{2}$. The triangle would then have a base of 3 and a height of 3. Therefore, the area of the triangle is $\frac{1}{2} \cdot 3 \cdot 3 = \frac{9}{2}$. Answer = $A + B + C + 2D = -3 + 2 - 1 + 2\left(\frac{9}{2}\right) = -2 + 9 = 7$
6	A=39 B=24 C=11 D=4 <u>Answer</u> 48	a) $((x^2)^3)^A = x^{234} \to x^{6A} = x^{234} \to 6A = 234 \to A = \frac{234}{6} = 39$ b) $\sqrt[4]{\sqrt[3]{y^B}} = y^2 \to y^{\frac{1}{3}\cdot\frac{1}{4}\cdot B} = y^2 \to y^{\frac{B}{12}} = y^2 \to \frac{B}{12} = 2 \to B = 24$ c) $\frac{z^4(z^3)^2}{z^C} = z^{-1} \to \frac{z^4 z^6}{z^C} = z^{-1} \to z^{10-C} = z^{-1} \to 10 - C = -1 \to C = 11$ d) $D^D = 2^8 \to D^D = (2^2)^4 \to D^D = 4^4 \to D = 4$ Answer = $(A + B) - (C + D) = (39 + 24) - (11 + 4) = 63 - 15 = 48$
7	A=15 B=12 C=50 D=30 <u>Answer</u> 107	a) Tim swims $\frac{1km}{18min}$. Since he runs three times faster, he will run $\frac{1km}{6min}$. Therefore, using proportions, $\frac{1}{6} = \frac{2.5}{x} \to x = 2.5 \cdot 6 = 15$ minutes b) Tom runs and swims at the same speed so he completed $\frac{3.5km}{42min}$. Therefore, using proportions, $\frac{3.5}{42} = \frac{1}{x} \to x = \frac{42}{3.5} = 12$ minutes c) Tam swims $\frac{1km}{10min}$. Since she swims twice as running, she will run $\frac{1km}{20min}$. Therefore, using proportions, $\frac{1}{20} = \frac{2.5}{x} \to x = 2.5 \cdot 20 = 50$ min. d) Tum runs at $\frac{2.5km}{30min}$. Since the rate of swimming is $\frac{2}{5}$ of the rate of running, the swimming speed is $\frac{2.5km}{30min} \cdot \frac{2}{5}$. Therefore, using proportions $\frac{5}{150} = \frac{1}{x} \to x = \frac{150}{5} = 30$ min Answer = $A + B + C + D = 15 + 12 + 50 + 30 = 107$
8	A=3 B=1 C=2 D=3	Plug in the coordinates (4,2) into the system #1: $4A + 2B = 14$ and $8 - 6 = C$ Plug in the coordinate $(5, -1)$ into the system #2: $5A - B = 14$ and $10 - 7 = D$

	Answer 18	To get parts a and b solve the system of equations with A and B: $4A + 2B = 14 \rightarrow \cdot 1 \rightarrow 4A + 2B = 14$ Same equations? How do you go to next line in solutions? $5A - B = 14 \rightarrow \cdot 2 \rightarrow 10A - 2B = 28$ Sum two equations to get $14A = 42 \rightarrow A = 3$ and after plugging it back in $B = 1$. Parts C and D are found by evaluate the earlier expressions, $C = 8 - 6 = 2$, $D = 10 - 7 = 3$. Answer =$A \cdot B \cdot C \cdot D = (3)(1)(2)(3) = 18$
9	A=9 B=8 C=5 D=-16 Answer 6	a) The y-intercept, plug in $x = 0$, $y = (0)^2 - 10(0) + 9$, $y = 9$ b) The x-intercepts can be found by factoring, $0 = (x - 9)(x - 1) \rightarrow x = 1,9$. The distance between the two x-intercepts is 8. c) The vertex x-value lies on the axis of symmetry, $x = -\frac{B}{2A} = -\frac{(-10)}{2(1)} = \frac{10}{2} = 5$ d) The y-value of the vertex is the minimum value, $(5)^2 - 10(5) + 9 = 25 - 50 + 9 = -16$ Answer = $A + B + C + D = 9 + 8 + 5 - 16 = 6$
10	A=-1 B=1 C=4/3 D=0 Answer 5	a) $1^2 - 2^1 = 1 - 2 = -1$ b) $2^1 - 1^2 = 2 - 1 = 1$ c) $3@ - 1 \rightarrow (3)^{-1} - (-1)^3 \rightarrow \frac{1}{3} - (-1) \rightarrow \frac{1}{3} + 1 = \frac{4}{3}$ d) $1@(2@3) \rightarrow 1@(2^3 - 3^2) \rightarrow 1@(8 - 9) \rightarrow 1@ - 1 \rightarrow 1^{-1} - (-1)^1 \rightarrow 1 - 1 = 0$ Answer = $A + 2B + 3C + 4D = -1 + 2(1) + 3\left(\frac{4}{3}\right) + 4(0) = -1 + 2 + 4 = 5$
11	A=12 B=15 C=13 D=14 Answer 54	a) $A + 10 = 2(A - 1) \rightarrow A + 10 = 2A - 2 \rightarrow A = 12$ b) $B = 1 + 2 + 3 + 4 + 5 = 15$ c) $C - 3 = \frac{2}{3}(C + 2) \rightarrow 3C - 9 = 2(C + 2) \rightarrow 3C - 9 = 2C + 4 \rightarrow C = 13$ d) $2D - 1 = 3(D - 5) \rightarrow 2D - 1 = 3D - 15 \rightarrow D = 14$ Answer = 12+15+13+14 = 54
12	A=16 B=25 C=10 D=20 Answer 90	a) 80% of 20 is $\frac{4}{5} \cdot 20 = \$16$ b) 80% of x is 20 $\rightarrow \frac{4}{5} \cdot x = 20 \rightarrow x = 20 \cdot \frac{5}{4} = \$25$ c) Organized counting technique table #1 (10 ways) d) Organized counting technique table #2 (20 ways)

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Answer = $(B-A)\cdot(D-C)=(25-16)(20-10)=(9)(10)=90$
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<td>A=2
B=0
C=9
D=3

<u>Answer</u>
144</td>
<td>a) $|x+2|=10\rightarrow x+2=10$ and $x+2=-10$. Both solutions are integers, $x=8,-12$
b) $8-2|x|=10\rightarrow -2|x|=2\rightarrow |x|=-1$. There are no real solutions as an absolute value will not give a negative answer.
c) $|x|\leq 4$, $x\leq 4$ and $x\geq -4$. There are 9 integral values, $x=-4,-3,-2,-1,0,1,23,4$
d) $|2x-8|<4\rightarrow 2x-8<4$ and $2x-8>-4$. When solved $x<6$ and $x>2$. There are only 3 possible integral values, $x=3,4,5$.

a) Answer = $(B+C+D)^A=(0+9+3)^2=12^2=144$</td>
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B=3
C=8
D=64

<u>Answer</u>
73</td>
<td>a) $2(3-4)^5\rightarrow 2(-1)^5\rightarrow 2(-1)\rightarrow -2$
b) $\frac{4+3^2-1}{2^3-4}\rightarrow\frac{4+9-1}{8-4}\rightarrow\frac{12}{4}\rightarrow 3$
c) $\sqrt{10^2-10(4)+4}\rightarrow\sqrt{100-40+4}\rightarrow\sqrt{64}\rightarrow 8$
d) $\frac{4^2}{2^{-2}}\rightarrow\frac{16}{\frac{1}{4}}\rightarrow 64$

Answer = $A+B+C+D=-2+3+8+64=73$</td>
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<tr>
<td>15</td>
<td>A=1
B=1
C=2
D=1

<u>Answer</u>
1121</td>
<td>b) Sometimes (1); For example $1+2$ is rational but $\sqrt{2}+1$ is irrational
c) Sometimes (1); For example $\sqrt{2}\cdot\sqrt{2}$ is rational but $\sqrt{2}\cdot 2$ is irrational
d) Always (2); For example $-3\cdot 2$ is always an integer (non-decimal value)
e) Sometimes (1); For example $\sqrt{2}^2$ is an integer but $\sqrt{2}^3$ is not an integer

Answer = $1000A+100B+10C+D=1000+100+20+1=1121$</td>
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