Pre-Calculus Team Solutions

1. $\mathbf{0}$

2. $\frac{58}{355}$

3. $64\sqrt{6}$

4. $\frac{13\sqrt{10}}{5}$

5. $12\sqrt{3} - 18$

6. $\frac{194}{95}$

7. 3

8. 22

9. $\frac{159}{2}$

10. $\frac{1-i}{7}$

11. 808

12. $\frac{640\sqrt{13}}{13}$

13. 4152

14. $\frac{25+25\sqrt{3}}{6}$

15. $\frac{56}{17}$

SOLUTION KEY- Part A, Part B, Part C, Part D

1. **The answer is 0**

$$x = \frac{g^{-1}(x)}{g^{-1}(x)+1}$$

$$(g^{-1}(x)+1)x - g^{-1}(x) = 0$$

$$[g^{-1}(x)][x-1] = -x$$

$$g^{-1}(x) = \frac{x}{1-x}$$

Set $g(x) = g^{-1}(x)$

$$\frac{x}{x+1} = \frac{x}{1-x}$$

$$1 + x = 1 - x$$

$$x = 0$$

$$g(0) = 0$$

$$n + m = 0$$

$$x = \ln(f^{-1}(x)+5) + 3$$

$$x - 3 = \ln(f^{-1}(x)+5)$$

$$f^{-1}(x) + 5 = e^{(x-3)}$$

$$f^{-1}(x) = e^{(x-3)} - 5$$

$$f^{-1}(3) = -4$$

$$-{}^{b}/_{a} = \frac{1}{2}$$

$$ABC = (0)(-4)\left(\frac{1}{2}\right)$$

$$ABC = 0$$

2. **The answer is** $\mathbf{\frac{58}{355}}$

The first digit can be 1-9 so there are 9 choices. The second digit can be 0-9 so there are 10 choices. The third digit can be 0-9 so there are 10 choices. Hence there are 900 five digit palindromes

On the interval $(0, n]$, there are $\left\lfloor \frac{n}{5} \right\rfloor$ multiples of 5.

$$\left\lfloor \frac{10000}{5} \right\rfloor - \left\lfloor \frac{1000}{5} \right\rfloor = 1800$$

$$p = \frac{1800}{9000} = \frac{18}{90} = \frac{1}{5}$$

$$31^2 = 961$$

$$90^2 = 8100$$

There are 58 perfect squares on the open interval $[1000,8100)$

$$p = \frac{58}{7100}$$

4+1, 1+4, 2+3, 3+2 There are 4 combinations

$$p = \frac{1}{9}$$

$$ABCD = (900)\left(\frac{58}{7100}\right)\left(\frac{1}{9}\right)\left(\frac{1}{5}\right)$$

$$ABCD = (100)\left(\frac{58}{7100}\right)\left(\frac{1}{5}\right)$$

$$ABCD = (20)\left(\frac{58}{7100}\right)$$

$$ABCD = \frac{(2)(58)}{710}$$

$$ABCD = \frac{58}{355}$$

3. **The answer is $64\sqrt{6}$**

$$\sqrt{8192} = \sqrt{64^2 \times 2}$$

$$\sqrt{8192} = 64\sqrt{2}$$

$$\left(\frac{\sqrt{3}+i}{2}\right)^{2020} = \left(cis\ \frac{\pi}{6}\right)^{2020}$$

$$\left(\frac{\sqrt{3}+i}{2}\right)^{2020} = \left(cis\ \frac{2020\pi}{6}\right)$$

Reduce by 336π

$$\left(\frac{\sqrt{3}+i}{2}\right)^{2020} = \left(cis\ \frac{2\pi}{3}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$Im\left(\left(\frac{\sqrt{3}+i}{2}\right)^{2020}\right) = \frac{\sqrt{3}}{2}$$

Each event is independent. The probability to land on heads is $\frac{1}{2}$

The Amplitude is 4

$$ABCD = \left(64\sqrt{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)(4)$$

$$ABCD = 64\sqrt{6}$$

4. **The answer is** $\frac{13\sqrt{10}}{5}$

$$5(x^2 - 6x) + 4(y^2 - 8y) = -69$$

$$5(x-3)^2 + 4(y-4)^2 = -69 + 45 + 64$$

$$\frac{(x-3)^2}{8} + \frac{(y-4)^2}{10} = 1$$

$$Length\ of\ Latus\ Rectum = \frac{2b^2}{a}$$

$$Length\ of\ Latus\ Rectum = \frac{2(8)}{\sqrt{10}} = \frac{8\sqrt{10}}{5}$$

$$S = \frac{a_1}{1-r}$$

$$S = \frac{\frac{3}{4}}{\left(1 - \left(-\frac{2}{3}\right)\right)} = \frac{9}{20}$$

$$y - k = \frac{1}{4p}(x-h)^2$$

$$p = \frac{1}{6}$$

$$S = \frac{n}{2}(a_0 + a_n)$$

$$S = \frac{13}{2}\left(\frac{1}{6} + \frac{38}{12}\right)$$

$$S = \frac{13}{2}\left(\frac{20}{6}\right) = \frac{65}{3}$$

$$ABCD = \left(\frac{8\sqrt{10}}{5}\right)\left(\frac{9}{20}\right)\left(\frac{1}{6}\right)\left(\frac{65}{3}\right)$$

$$ABCD = \left(\frac{8\sqrt{10}}{5}\right)\left(\frac{1}{20}\right)\left(\frac{1}{2}\right)(65)$$

$$ABCD = (4\sqrt{10})\left(\frac{1}{20}\right)(13)$$

$$ABCD = \frac{13\sqrt{10}}{5}$$

5. **The answer is $12\sqrt{3} - 18$**

$$\cos 3x = \frac{\sqrt{3}}{2}$$

$$3x = \pm\frac{\pi}{6} + 2\pi n$$

$$x = \left\{\frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}\right\}$$

$$\frac{\pi}{18} + \frac{11\pi}{18} + \frac{13\pi}{18} = \frac{25\pi}{18}$$

$$\tan 15° = \frac{\sin 30°}{1 + \cos 30°}$$

$$\tan 15° = \frac{1/2}{1 + \sqrt{3}/2}$$

$$\tan 15° = \frac{1}{2 + \sqrt{3}}$$

$$\tan 15° = \frac{1}{2 + \sqrt{3}}\left(\frac{2 - \sqrt{3}}{2 - \sqrt{3}}\right)$$

$$\tan 15° = 2 - \sqrt{3}$$

$$P = \frac{2\pi}{\left(\frac{1}{5}\right)} = 10\pi$$

$$A = \frac{1}{2}(CD)(CE)\sin 60°$$

$$A = (0.5)(4)(5)\left(\sqrt{3}/2\right)$$

$$A = 5\sqrt{3}$$

$$\frac{C}{6A} - BD = \frac{10\pi}{6\left(\frac{25\pi}{18}\right)}(2 - \sqrt{3})(5\sqrt{3})$$

$$\frac{C}{6A} - BD = \frac{10\pi}{\left(\frac{25\pi}{3}\right)}(10\sqrt{3} - 15)$$

$$\frac{C}{6A} - BD = \frac{6}{5}(10\sqrt{3} - 15)$$

$$\frac{C}{6A} - BD = 12\sqrt{3} - 18$$

6. **The answer is $\frac{194}{95}$**

$$f(2) = 4 \rightarrow g(4) = 75$$

$$A = \frac{1}{2}(10)(5) = 25$$

$$\frac{AC}{B} = \left(\frac{75}{25}\right)\left(\frac{194}{285}\right)$$

$$\frac{AC}{B} = \frac{194}{95}$$

The defect rate is 30% → six defective transducers in the shipment

The probability the first transducer is not defective is $\frac{14}{20}$

The probability the second transducer is not defective is $\frac{13}{19}$

The probability the third transducer is not defective is $\frac{12}{18}$

$$p(at\ least\ one\ defective) = 1 - p(none\ are\ defective)$$

$$P(none\ are\ defective) = 1 - \left(\frac{14}{20}\right)\left(\frac{13}{19}\right)\left(\frac{12}{18}\right) = \frac{194}{285}$$

7. **The answer is 3**

Let $F(x) = Ax^4 + Bx^3 + Cx^2 + Dx$

Condition I: $f(-1) = 3 \rightarrow A - B + C - D = 3$

Condition II : $f(-2) = 2 \rightarrow 16A - 8B + 4C - 2D = 2$

Condition III: $f(1) = -3 \rightarrow A + B + C + D = -3$

Condition IV: $f(2) = 4 \rightarrow 16A + 8B + 4C + 2D = 4$

Add Conditions I and III → $2A + 2C = 0 \rightarrow A + C = 0$

Add Conditions II and IV → $32A + 8C = 6$

$32A + 8C = 6 \rightarrow 24A + (8A + 8C) = 6 \rightarrow 24A = 6 \rightarrow A = {}^{1}/_{4}$

$$\frac{88-\mu}{\sigma} = \frac{7}{3}$$

$$\frac{92-\mu}{\sigma} = \frac{8}{3}$$

$$\frac{92-\mu}{\sigma} = \frac{88-\mu}{\sigma} + \frac{1}{3}$$

$$AB = \left(\frac{1}{4}\right)(12)$$

$$AB = 3$$

8. **The answer is 22**

$$m_1 = \frac{1}{2} \qquad m_2 = \frac{1}{3}$$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{{}^1\!/_3 - ({}^1\!/_2)}{1 + ({}^1\!/_2)({}^1\!/_3)}\right|$$

$$\tan\theta = \left|\frac{-{}^1\!/_6}{1 + ({}^1\!/_2)({}^1\!/_3)}\right|$$

$$\tan\theta = \frac{1}{7}$$

$$\lim_{x\to 4}\left(\frac{x^2 - 3x - 10}{x^2 + 6x + 8}\right) = \lim_{x\to 4}\left(\frac{x-5}{x+4}\right)$$

$$\lim_{x\to 4}\left(\frac{x^2 - 3x - 10}{x^2 + 6x + 8}\right) = -\frac{1}{8}$$

$$det(AB) = det(A) + det(B)$$

$$\begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix} = 18 - 20 = -2$$

$$\begin{vmatrix} 8 & 9 \\ 5 & 7 \end{vmatrix} = 56 - 45 = 11$$

$$det(AB) = -22$$

$$\langle 5,9,-3\rangle \bullet \langle -5,7,-6\rangle = -25 + 63 + 18$$

$$\langle 5,9,-3\rangle \bullet \langle -5,7,-6\rangle = 56$$

$$ABCD = \left(\frac{1}{7}\right)\left(\frac{-1}{8}\right)(-22)(56)$$

$$ABCD = 22$$

9. **The answer is $\frac{159}{2}$**

$$\begin{bmatrix} 3 & 5 \\ -4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ -44 \end{bmatrix}$$

$$3x + 5y = 14 \ \ (\times 4)$$

$$-4x + 6y = -44 \ \ (\times 3)$$

$$38y = -76$$

$$y = -2$$

$$x = \frac{1}{3}\big(14 - 5(-2)\big) = 8$$

$$x + y = 6$$

$$A + B + C = 6 + 92 - \frac{37}{2}$$

$$A + B + C = 98 - \frac{37}{2}$$

$$A + B + C = \frac{159}{2}$$

$$\begin{vmatrix} 2 & -3 & 2 \\ 5 & 1 & 7 \\ 3 & -2 & 9 \end{vmatrix} = 2\begin{vmatrix} 1 & 7 \\ -2 & 9 \end{vmatrix} - (-3)\begin{vmatrix} 5 & 7 \\ 3 & 9 \end{vmatrix} + 2\begin{vmatrix} 5 & 1 \\ 3 & -2 \end{vmatrix}$$

$$\begin{vmatrix} 2 & -3 & 2 \\ 5 & 1 & 7 \\ 3 & -2 & 9 \end{vmatrix} = 2(9 + 14) - (-3)(45 - 21) + 2(-10 - 3)$$

$$\begin{vmatrix} 2 & -3 & 2 \\ 5 & 1 & 7 \\ 3 & -2 & 9 \end{vmatrix} = 46 + 72 - 26 = 92$$

$$\begin{vmatrix} 3 & 4 & 1 \\ -2 & -5 & 1 \\ 6 & 2 & 1 \end{vmatrix} = 3\begin{vmatrix} -5 & 1 \\ 2 & 1 \end{vmatrix} - 4\begin{vmatrix} -2 & 1 \\ 6 & 1 \end{vmatrix} + \begin{vmatrix} -2 & -5 \\ 6 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 4 & 1 \\ -2 & -5 & 1 \\ 6 & 2 & 1 \end{vmatrix} = 3(-5 - 2) - 4(-2 - 6) + (-4 + 30)$$

$$\begin{vmatrix} 3 & 4 & 1 \\ -2 & -5 & 1 \\ 6 & 2 & 1 \end{vmatrix} = -21 + 32 + 26 = 37$$

$$A = \pm\frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ -2 & -5 & 1 \\ 6 & 2 & 1 \end{vmatrix} = \frac{37}{2}$$

10. **The answer is $\frac{1-i}{7}$**

$$f(x) = (x-3)(x+2)(x+2)$$

$$(3)(-2)(-2) = 12$$

$$\sum_{n=1}^{\infty}\left(\frac{i}{i-1}\right)^{n-1}$$

$$sum = \frac{a_1}{1-r} = \frac{1}{1-\left(\frac{i}{i-1}\right)}$$

$$sum = \frac{i-1}{i-1-i} = 1-i$$

$$|9+12i| = \sqrt{81+144} = 15$$

$$\frac{7!}{2!\,2!} = 1260$$

$$\frac{ABC}{D} = \frac{(12)(1-i)(15)}{1260}$$

$$\frac{ABC}{D} = \frac{1-i}{7}$$

11. **The answer is 808**

$|\cos(x)| < \frac{1}{2}$ on the following intervals $\left(-\frac{2\pi}{3}, -\frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$

$$p = \frac{\left|-\frac{\pi}{3} - \left(-\frac{2\pi}{3}\right)\right| + \left|\frac{2\pi}{3} - \left(\frac{\pi}{3}\right)\right|}{\left|\frac{2\pi}{3} - \left(-\frac{5\pi}{6}\right)\right|}$$

$$p = \frac{\frac{\pi}{3} + \frac{\pi}{3}}{\frac{3\pi}{2}} = \frac{4}{9}$$

Must travel 12 units vertically and 4 units horizontally. Finding the number of paths to travel vertically is sufficient because for each unique vertical path is a corresponding unique horizontal path. Each combination of vertical and horizontal displacements is unique.

$$\binom{16}{12} = \frac{16!}{12!\,4!} = \frac{16 \times 15 \times 14 \times 13}{4 \times 3 \times 2}$$

$$\binom{16}{12} = 4 \times 5 \times 7 \times 13 = 1820$$

$$A(B-2) = \frac{4}{9} \times 1818$$

$$A(B-2) = 808$$

12. **The answer is $\frac{640\sqrt{13}}{13}$**

$$\cos\left(Arccot\left(\frac{5}{12}\right)\right) = \frac{5}{13}$$

$$\left(4, \frac{\pi}{3}\right) \to 4cis\left(\frac{\pi}{3}\right) = 2 + 2\sqrt{3}i$$

$$\left(6\sqrt{3}, \frac{\pi}{2}\right) \to 6\sqrt{3}cis\left(\frac{\pi}{2}\right) = 6\sqrt{3}i$$

$$d = \sqrt{2^2 + \left(6\sqrt{3} - 2\sqrt{3}\right)^2}$$

$$d = \sqrt{52} = 2\sqrt{13}$$

$$x^2 + y^2 = 64$$

$$A = 64\pi$$

$$\frac{ABC}{\pi} = \frac{5}{13} \times 2\sqrt{13} \times \frac{64\pi}{\pi}$$

$$\frac{ABC}{\pi} = \frac{10}{\sqrt{13}} \times 64$$

$$\frac{ABC}{\pi} = \frac{640\sqrt{13}}{13}$$

13. **The answer is 4152**

$$r = -\frac{2}{5}$$

$$S = \frac{a_1}{1-r} = \frac{\left({}^{1}/{}_{3}\right)}{1 - \left(-{}^{2}/{}_{5}\right)}$$

$$S = \frac{5}{15 - (-6)} = \frac{5}{21}$$

The common difference is 4

$$a_{100} = 2 + (11 - 1)(4) = 42$$

$$72^{11} = (2 \times 6^2)^{11}$$

$$72^{11} = 2^{33} \times 3^{22}$$

$$\#\ of\ Divisors = (33 + 1)(22 + 1) = 782$$

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

$$\binom{10}{k} (x^2)^k \left(\frac{2}{x^3}\right)^{10-k}$$

$$2k = 3(10-k)$$

$$k = 6$$

$$\binom{10}{6} (x^2)^6 \left(\frac{2}{x^3}\right)^{10-6} = 3360$$

$$AB + C + D = \binom{5}{21}(42) + 782 + 3360$$

$$AB + C + D = 10 + 782 + 3360$$

$$AB + C + D = 4152$$

14. **The answer is** $\frac{\mathbf{25+25\sqrt{3}}}{\mathbf{6}}$

$$\lim_{x\to 2}\left(\sqrt{e^{(10+4\ln(x))}}\right) = \sqrt{e^{(10+\ln(2^4))}} = 4e^5$$

$$\lim_{x\to \pi/3}\left(\frac{\tan^2 x}{\sec x}\right) = \frac{\tan^2(\pi/3)}{\sec(\pi/3)} = \frac{3}{(2)} = \frac{3}{2}$$

$$.\ \lim_{x\to\infty}\left(\frac{3x^4-2x^3+6}{10x^4-7x^3-3x}\right) = \frac{3}{10}$$

$$\lim_{x\to \pi/6}\left(\frac{\sin x + \cos x}{\cot^2 x}\right) = \frac{\sin(\pi/6) + \cos(\pi/6)}{\cot^2(\pi/6)}$$

$$\lim_{x\to \pi/6}\left(\frac{\sin x + \cos x}{\cot^2 x}\right) = \frac{1/2 + \left(\sqrt{3}/2\right)}{3} = \frac{1}{6}(1+\sqrt{3})$$

$$\frac{BD}{C} ln\left(\frac{A}{4}\right) = \frac{\left(\frac{3}{2}\right)\left(\frac{1+\sqrt{3}}{6}\right)}{\left(\frac{3}{10}\right)} ln\left(\frac{4e^5}{4}\right) = \frac{\left(\frac{1}{2}\right)\left(\frac{1+\sqrt{3}}{2}\right)}{\left(\frac{3}{10}\right)}(5)$$

$$\frac{BD}{C} ln\left(\frac{A}{4}\right) = \frac{10(1+\sqrt{3})}{4(3)}(5)$$

$$\frac{BD}{C} ln\left(\frac{A}{4}\right) = \frac{5(1+\sqrt{3})}{2(3)}(5) = \frac{25+25\sqrt{3}}{6}$$

15. **The answer is** $\frac{56}{17}$

There are 20 marbles in total

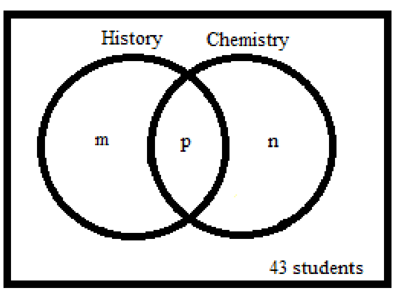
$$p = \frac{\binom{6}{4}\binom{4}{4}\binom{8}{6}\binom{2}{0}}{\binom{20}{16}}$$

$$\binom{2}{0} = 1 \qquad \binom{4}{4} = 1 \qquad \binom{6}{4} = 15$$

$$\binom{8}{6} = 28 \qquad \binom{20}{16} = 4845$$

$$p = \frac{15 \times 28}{4845} = \frac{28}{323}$$

$$AB = \frac{28}{323}(38) = \frac{56}{17}$$



$$m + p + n + 43 = 175$$

$$m + p = 94$$

$$n = 38$$