

The acronym NOTA denotes that “none of the above answers is correct.” All answers are rationalized, simplified, and exact unless otherwise stated. Let $\text{cis } \theta = \cos \theta + i \sin \theta$, $z = a + bi$, and $i = \sqrt{-1}$.

1. Evaluate $\lim_{n \rightarrow \infty} \frac{n}{e^n}$

- A. ∞ B. $\ln x$ C. 1 D. 0 E. NOTA

2. An ASML plant manufactures computer chips independently with a defect rate of 20%. Five computer chips are tested, before leaving the factory, in a randomly selected group of ten computer chips. What is the probability that at least one chip will be defective?

- A. $\frac{11}{12}$ B. $\frac{8}{9}$ C. $\frac{7}{9}$ D. $\frac{2}{9}$ E. NOTA

3. Since 2016, the student population at Deerfield Beach High has continued to decline according the equation: $P(x) = 2700 - \frac{1800(x-2016)}{e^{\ln(x-2013)}}$, where x is the year and P is the student population. The City of Deerfield Beach statues mandate if a school census returns a population of less than 1500, then the school should be shut down. The census is taken once a year in November. Decisions to stay open or to close are made immediately after the census is taken. During which year will the school shut down?

- A. 2023 B. 2022 C. 2018 D. 2031 E. NOTA

4. A sequence is defined recursively by $2a_{n+1}^2 = \sqrt{11a_n + \frac{6}{\sqrt{11a_n + \frac{6}{\dots}}}}$ for $n \geq 0$. Which of the following is equivalent to $\lim_{n \rightarrow \infty} a_n$ assuming a_n is a real number for each positive integer n ?

- A. $-\frac{1}{2}$ B. $\sqrt[3]{3}$ C. $\sqrt[3]{\frac{3}{4}}$ D. $-\sqrt[3]{\frac{1}{4}}$ E. NOTA

5. Which of the following is equivalent to $\frac{5-3i}{4+2i}$?

- A. $\frac{7+i}{10}$ B. $\frac{7-11i}{5}$ C. $\frac{13-i}{10}$ D. $\frac{7-11i}{10}$ E. NOTA

6. A surveyor is mapping a new park. She starts at Lamp post A. She walks 25 meters due southeast to Lamp post C. She then turns 60° counter-clockwise and walks 40 meters to Lamp post B. She concludes by walking directly back to Lamp post A. How far must she walk from Lamp post B to reach Lamp post A?

- A. $5\sqrt{69}$ B. $5\sqrt{129}$ C. 35 D. $5\sqrt{49 - 40\sqrt{3}}$ E. NOTA

7. A box of pens contains fourteen acceptable pens, four faulty pens (*fail immediately*), and six partially faulty pens (*will fail after one hour*). Faulty and partially faulty pens are considered unacceptable. A pen is chosen at random from the box and immediately used. If the chosen pen does not fail immediately, what is the probability that the pen is unacceptable?

- A. $\frac{1}{5}$ B. $\frac{3}{10}$ C. $\frac{9}{25}$ D. $\frac{3}{7}$ E. NOTA

8. Which of the following represents the equation of the line tangent to $f(x) = 4x^3 - 6x^2 + 2$ at $x = 2$?

- A. $y = 24x - 58$ B. $y = 12x - 14$ C. $y = 24x - 38$ D. $y = 36x - 62$ E. NOTA

9. Find the domain of the function $f(x) = \sqrt{2 - \log_6(x - 5)}$

- A. (5,41] B. [6,42) C. (5,105] D. [6,95) E. NOTA

10. Find the number of zeros at the end of the expansion of 1995!

- A. 498 B. 481 C. 417 D. 496 E. NOTA

11. Which of the following is the measure of the acute angle formed by the vectors $\langle 5, 1, -7 \rangle$ and $\langle 8, 2, 2 \rangle$?

- A. $\text{Cos}^{-1}\left(\frac{7\sqrt{6}}{45}\right)$ B. $\text{Cos}^{-1}\left(\frac{4\sqrt{6}}{15}\right)$ C. $\text{Cos}^{-1}\left(\frac{14\sqrt{6}}{45}\right)$ D. $\text{Cos}^{-1}\left(\frac{7\sqrt{10}}{45}\right)$ E. NOTA

12. Compute $\begin{vmatrix} -1 & 4 & 6 \\ 1 & 3 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

- A. 8 B. 30 C. 25 D. 37 E. NOTA

20. Find the quotient $\frac{z_1}{z_2}$ of the complex numbers

$$z_1 = 4cis \frac{5\pi}{4}$$

$$z_2 = 6cis \frac{\pi}{12}$$

- A. $\frac{i\sqrt{3}-3}{4}$ B. $\frac{3-i\sqrt{3}}{3}$ C. $\frac{-\sqrt{3}-i}{3}$ D. $\frac{1+i\sqrt{3}}{2}$ E. NOTA

21. Let $\vec{u} = \langle 4, -2, 7 \rangle$ and $\vec{v} = \langle -5, 3, -6 \rangle$. Find $\vec{u} \times \vec{v}$ in the form $\langle a, b, c \rangle$

- A. $\langle 9, 59, -2 \rangle$ B. $\langle -9, 11, 2 \rangle$ C. $\langle -33, 11, 22 \rangle$ D. $\langle -9, -11, 2 \rangle$ E. NOTA

22. Solve for $3A + B + C$: $\frac{6x^2+4x-6}{(x-3)(x^2+2)} = \frac{Ax+B}{x^2+2} + \frac{C}{x-3}$

- A. 15 B. 13 C. 16 D. 10 E. NOTA

23. How many distinct ways can you arrange 9 people around a circular table?

- A. 3024 B. 362880 C. 40320 D. 10080 E. NOTA

24. Which of the following best describes the graph $r = 5 - 4 \cos \theta$

- A. Lemniscate B. Limaçon without loop C. Limaçon with loop D. Rose Curve E. NOTA

25. Find the distance between the point $\langle 4, 2, 4 \rangle$ and the plane $5x - 4y + 3z = 12$

- A. $\frac{6\sqrt{5}}{5}$ B. 6 C. $\frac{6\sqrt{2}}{5}$ D. $\frac{18\sqrt{2}}{5}$ E. NOTA

26. Solve for x , given $x > 0$: $\log_5 x^2 + \log_5 9 = 4$

- A. $\frac{3}{25}$ B. $\frac{100}{3}$ C. $\frac{2}{3}$ D. $\frac{25}{3}$ E. NOTA

27. In Snow's class, thirty-four students take History, thirty-one students take Art, and twenty-seven students take Chemistry. There are only twelve students taking History and Chemistry, only thirteen students taking Art and History, and only two students taking all three courses. All fifty-six students take History, Art or Chemistry. What is the probability a randomly chosen student in Snow's class takes only Art?

- A. $\frac{1}{8}$ B. $\frac{1}{7}$ C. $\frac{3}{14}$ D. $\frac{1}{4}$ E. NOTA

28. Evaluate $\lim_{x \rightarrow \pi} \left(\frac{x}{\cos x} \right)$

- A. 1 B. $-\pi$ C. π D. -1 E. NOTA

29. Find $\begin{bmatrix} 4 & 8 \\ 2 & 6 \end{bmatrix}^{-1}$

- A. $\begin{bmatrix} -\frac{1}{2} & \frac{1}{4} \\ 1 & -\frac{3}{4} \end{bmatrix}$ B. $\begin{bmatrix} \frac{1}{10} & -\frac{1}{5} \\ \frac{1}{20} & \frac{3}{20} \end{bmatrix}$ C. $\begin{bmatrix} \frac{3}{4} & -1 \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$ D. $\begin{bmatrix} \frac{3}{10} & -\frac{4}{10} \\ -\frac{1}{10} & \frac{1}{5} \end{bmatrix}$ E. NOTA

30. Find the unit vector in the same direction as $\langle 5, 4, 2 \rangle$

- A. $\langle \frac{\sqrt{5}}{9}, \frac{4\sqrt{5}}{45}, \frac{2\sqrt{5}}{45} \rangle$ B. $\langle \frac{5\sqrt{11}}{22}, \frac{2\sqrt{11}}{11}, \frac{\sqrt{11}}{11} \rangle$ C. $\langle \frac{5\sqrt{11}}{11}, \frac{4\sqrt{11}}{11}, \frac{2\sqrt{11}}{11} \rangle$ D. $\langle \frac{\sqrt{5}}{3}, \frac{4\sqrt{5}}{15}, \frac{2\sqrt{5}}{15} \rangle$ E. NOTA