

GEOMETRY INDIVIDUAL TEST

March 2021

Florida Regional Competition

The abbreviation "NOTA" means "None Of The Above."
 Diagrams may not be drawn to scale.
 All angle measures are degrees.
 Reference to any polygon angle is assumed to be the interior angle, unless otherwise stated.

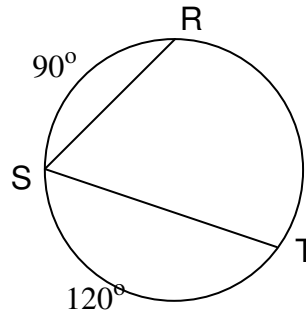
1. $\angle R$ has a complement that measures thirty more degrees than twice the measure of $\angle R$. Give the measure of $\angle R$.
 A. 70° B. 50°
 C. 24° D. 20°
 E. NOTA

2. \overline{RS} is perpendicular to \overline{ST} . \overline{ST} is parallel to \overline{RP} . Which of the following **may be true**?
 I. \overline{RS} is skew to \overline{RP} .
 II. \overline{RP} , \overline{ST} and \overline{RS} are coplanar.
 III. Two of the lines mentioned are skew.
 A. I only B. I, II only
 C. II only D. II, III only
 E. NOTA

3. $\triangle RST$ has a right angle at S. \overline{SP} is a median to \overline{RT} with P on \overline{RT} . If $SR=6$ and $ST=8$ then find PT.
 A. 5 B. $5\sqrt{3}$
 C. 5.4 D. $2\sqrt{5}$
 E. NOTA

4. $\triangle RST$ has a right angle at S. \overline{SP} is an altitude to \overline{RT} with P on \overline{RT} . If $SR=6$ and $ST=8$ then find SP.
 A. 3.6 B. 4.8
 C. 5 D. 6.4
 E. NOTA

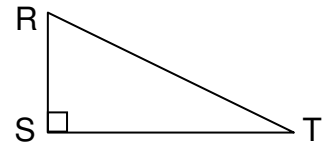
5.



In the circle above, which has radius 12 cm, $m\widehat{SR} = 90^\circ$ and $m\widehat{ST} = 120^\circ$.

- Find the length in cm of minor arc \widehat{RT} .
- A. 10π B. 11π
 C. 120π D. 150π
 E. NOTA

6. In right $\triangle RST$, with right angle at S,

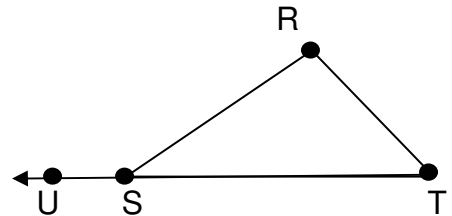


$$\sin T = \frac{x}{x+8} \text{ and } \cos T = \frac{x+7}{x+8}.$$

Which gives $\tan T$?

- A. $\frac{1}{2}$ B. $\frac{\sqrt{113}}{15}$
 C. $\frac{5}{12}$ D. $\frac{6}{13}$
 E. NOTA

7.

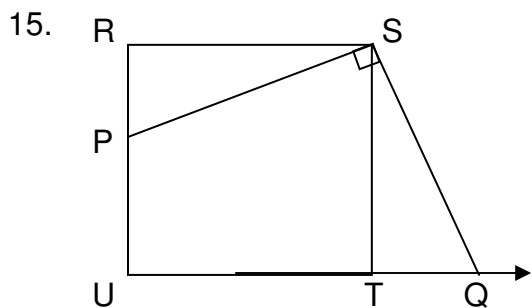


Given $\triangle RST$ with exterior angle at S, shown above, $m\angle USR = (2x)^\circ$,

$$m\angle T = \left(\frac{1}{2}x\right)^\circ \text{ and } m\angle R = (x+40)^\circ.$$

Give the value of x .

- A. 80 B. 78
 C. 76 D. 20
 E. NOTA



Square RSTU has area 64 sq. cm. Point P is on side \overline{RU} . The segment perpendicular to \overline{PS} is drawn at S so that it intersects with the line which contains \overline{UT} . That segment, \overline{SQ} , intersects \overline{UT} at Q. The area of $\triangle PSQ$ is 50 sq. cm. Give the length of \overline{TQ} in cm.

- A. 6
- B. $4\sqrt{3}$
- C. 4
- D. $2\sqrt{3}$
- E. NOTA

16. The shortest leg of a right triangle has length 12. The hypotenuse has length 24. Which of the following must be true about the triangle?
- A. The angles are $30^\circ, 60^\circ$ and 90° .
 - B. The angles are congruent to that of a triangle with sides 3, 4 and 5.
 - C. The side lengths are all integers.
 - D. The triangle is isosceles.
 - E. NOTA

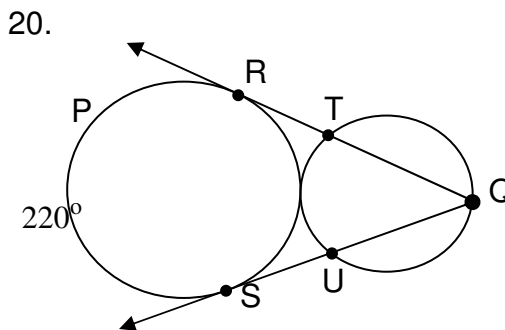
17. A cone has volume 100 cubic cm. A similar cone has radius that is half the radius of the first cone. What is the volume of the smaller cone, in cubic cm?
- A. 50
 - B. 25
 - C. 12.5
 - D. 6.25
 - E. NOTA

18. A rhombus has area 50 and side lengths each 10. What is the product of the diagonals of the lengths of the rhombus?

- A. $100\sqrt{3}$
- B. 100
- C. $50\sqrt{3}$
- D. 50
- E. NOTA

19. A regular hexagon has side lengths 10 each. What is the area of the hexagon?

- A. $150\sqrt{3}$
- B. $96\sqrt{3}$
- C. $90\sqrt{3}$
- D. $80\sqrt{3}$
- E. NOTA

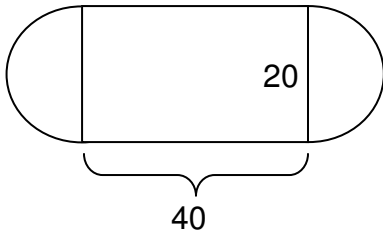


The circles above are externally tangent to each other. Q, T and U are on the smaller circle. \overline{QT} and \overline{QU} are tangent to the larger circle at points R and S respectively. $m\widehat{RPS} = 220^\circ$. R, T and Q are collinear.

Find $m\widehat{TU}$.

- A. 60°
- B. 65°
- C. 70°
- D. 80°
- E. NOTA

21.



A non-standard running track has a 40 meter by 20 meter rectangle in the center, with a semi-circle at each end, as shown. The perimeter of the track is $a + b\pi$ meters. Give the value of $a + b$.

- A. 900
- B. 320
- C. 120
- D. 100
- E. NOTA

22. A man starts at point S, walks m miles due east ($\xrightarrow{\text{that way}}$). He then is at point P and turns counter-clockwise 150° and walks 6 miles in the new direction, to point Q, which is exactly $2\sqrt{3}$ miles from his original position S, then find the value of m .

- A. $\sqrt{3}$
- B. $2\sqrt{3}$
- C. $3\sqrt{3}$
- D. $4\sqrt{3}$
- E. NOTA

23. A cooking dish is a right circular cylinder on the outside. The inside is half of a sphere.

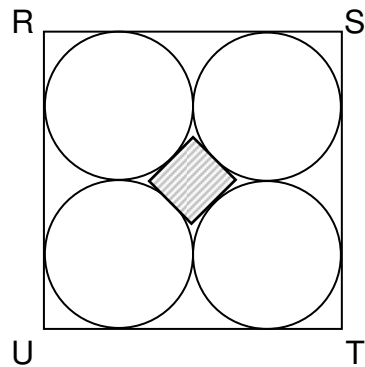


Find the volume of the dish (outside of the hemisphere and inside of the cylinder) in cubic cm, if the radius of the cylinder, which is the same as the radius of the hemi-sphere) is 6 cm.

- A. 152π
- B. 144π
- C. 72π
- D. 64π
- E. NOTA

24. In regular hexagon ABCDEF, opposite sides \overline{AB} and \overline{ED} are a distance of 90 cm apart. What is the length of one side of the hexagon in cm?

- A. $30\sqrt{6}$
- B. $30\sqrt{3}$
- C. $15\sqrt{6}$
- D. 30
- E. NOTA



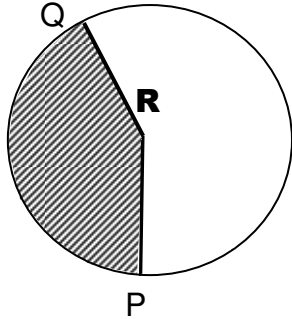
25. Square RSTU above has $RS=12$. Four congruent circles are each tangent to two sides of RSTU, and to two other circles as shown. A smaller square has sides tangent to each of the four circles, and the center of the small square is also center of the large square. Tell the length of one side of the smaller square.

- A. $4\sqrt{2}$
- B. $\frac{1}{2}(3\sqrt{2} + 3)$
- C. $3\sqrt{2}$
- D. $6\sqrt{2} - 6$
- E. NOTA

26. An equilateral triangle and a square have the same length side. Tell the ratio of the area of the triangle to the square.

- A. 1:2
- B. $1:2\sqrt{6}$
- C. $\sqrt{2}:\sqrt{3}$
- D. $2\sqrt{2}:\sqrt{3}$
- E. NOTA

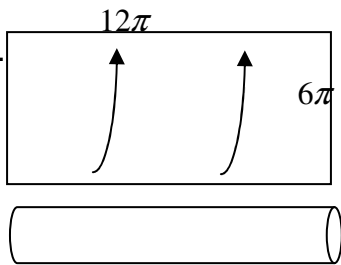
27.



A sector of a circle R of radius 12 is shaded above. The sector is 150° . Points P and Q are the endpoints of the minor arc that bounds the sector (with two radii). The non-shaded sector is cut out and radii \overline{RP} and \overline{RQ} are pulled together so that they coincide, to make a cone's lateral surface. Tell the volume of the cone with the shaded lateral surface.

- A. $40\sqrt{5}\pi$ B. $\frac{25\sqrt{119}}{3}\pi$
 C. $8\sqrt{119}\pi$ D. $\frac{325}{3}\pi$
 E. NOTA

28.



A rectangular piece of paper has dimensions 12π cm by 6π cm

The two long sides are brought together and a right circular cylinder is formed, with no bases. If the interior is then completely filled with clay, how many cubic cm of clay will fit? (Assume no overlap of surface area.)

- A. $60\pi^2$ B. $72\pi^2$
 C. $90\pi^2$ D. $108\pi^2$
 E. NOTA

29. Joe wanted to take a fishing pole on the bus. The rules say that any objects on the bus (except people) must have a maximum of 5 feet for length, 4 feet for width, and 1 foot for depth. $length \geq width \geq depth$. If we assume that his fishing pole is a cylinder and we ignore radius of his pole, then tell which is the longest pole that will fit in a box of the above dimensions, in feet.

- A. $\sqrt{42}$ B. $\sqrt{41}$
 C. $\sqrt{20}$ D. $\sqrt{19}$
 E. NOTA

30. Consider the true statement "If Joe passes this test, then Mark will be happy for him." If Joe does not pass this test, what must be true?

- A. Mark will be happy for him.
 B. Mark will not be happy for him.
 C. If Mark is happy for him then Joe did not tell him the score.
 D. If Mark is not happy for him then Joe lied to him.
 E. NOTA