1. $i=\sqrt{-1}$.

A = the value of $(a+b)$ for $\dfrac{4-i}{5+3i}=a+bi$.

B = the value of $(c+d)$ for $(3-4i)(c+di)=(-33-56i)$

C = the 4th term of the expansion of $(2x-y)^8$, if the expansion is $n_1x^8+n_2x^7+...+n_9$.

D = the sum of the coefficients of the expansion of $(x-2y)^9$.

Give the value of A+B+C+D for $x=1$ and $y=-1$.

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2. Let G_1 be the graph of $x=2y^2-4y+1$.

Let G_2 be the graph of $x^2-4y^2-4x+32y-64=0$.

The coordinates of the focus of G_1 are (A, B).

The absolute value of the slope of an asymptote to the graph of G_2 is C.

The distance between the foci of the graph of G_2 is D.

Give the value of 16ABCD.

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3. E_1 is the equation $(x-1)^{\frac{4}{3}}-2(x-1)^{\frac{2}{3}}=8$.

E_2 is the equation $\dfrac{3}{2}(x-1)^{\frac{4}{3}}=24$

E_3 is the equation of $2\left(\sqrt[3]{3x-1}\right)=-1$

A = the least integral solution of E_1.

B = the greatest real solution of E_1.

C = the greatest real solution of E_2.

D = the real solution of E_3.

Give the value of $\dfrac{\text{ABC}}{\text{D}}$.

---

4. $f(x)=\frac{x^2-3x-4}{16-x^2}$ and $g(x)=\frac{2}{x-3}-\frac{1}{3x+1}$.

A = the x-intercept of the vertical asymptote of the graph of f.
B = the x-intercept of the graph of f.
C = the least value of x for which $g(x)=-\frac{5}{4}$.
D = $x-$intercept of the graph of g.

Give the value of ABCD in fraction form.

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5. $A=\sum_{n=1}^{1000}\frac{3}{4}\left(\frac{1}{4}\right)^{n-1}$ rounded to the nearest whole number.

$B=$ the one-hundredth term of the sequence $\frac{3}{4},-\frac{4}{5},\frac{5}{6},-\frac{6}{7},\ldots$

$C=$ the value of x for which the series $2-4x+8x^2-16x^3+\ldots$ is equal to $\frac{4}{3}$.

$D=$ the sum of the first 101 terms of the sequence $2,9,16,23,\ldots$

Give the value of $412(ABC)+D$.

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6. A: the solution to $4\left(\frac{1}{8}\right)^{A+2}=2^A$. B: the real solution to $\sqrt{1+\sqrt{B}}=\frac{\sqrt{5}}{2}$.

$D=$ the number of positive integral factors to the solution C of $\log_2\left(\log_3\left(\log_4 C\right)\right)=2$.

Give the value of $A+D+\frac{1}{B}$.

----

7. A = $C(12,3)$, the number of combinations of 12 objects taken 3 at a time.
B = $P(9,3)$ divided by $P(10,3)$, where $P(n,r)$ is the number of permutations of n objects taken r at a time.
D = positive integer value of k so that $\frac{k!(k+1)!}{(k-1)!(k+2)!}=\frac{2}{3}$.

Give the value of ABD.

8.

$E_1 \quad x^2+(y-3)^2=25$	$E_2:\ 4x+5y=-1$
$E_3:\ x^2+4y^2=16$	$E_4:\ x-2y=4$
$E_5:\ y=x-2$	

A = the greatest x-coordinate of the points where the graphs of E_1 and E_3 intersect.
B = the x-coordinate of the points on the graph of E_2 and E_5 intersect.
C = the number of intersection points for the graphs of E_3 and E_5.
D = the number of intersection points for the graphs E_5 and E_4.

Give the sum A+B+C+D.

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9. A jar has 5 blue marbles and 4 red marbles. The marbles are identical except for color, and are well mixed.

A = the probability that a marble is randomly drawn from the jar and that it is red. Let your answer be equal to $\frac{a}{b}$ in reduced form.
B = the probability that two marbles are randomly drawn at the same time from the jar, and they are both blue. Let your answer be equal to $\frac{c}{d}$ in reduced form.
C = the probability that two marbles are randomly drawn from the jar one after the other, with replacement, and they have different color (from each other). Let your answer be equal to $\frac{f}{g}$ in reduced form.
D = the probability that 5 marbles are randomly drawn from the jar without replacement, one after each other, and all are blue. Let your answer be equal to $\frac{h}{k}$ in reduced form.

For parts A, B, C and D written in reduced fraction form, your final answer is the sum of the numerators and denominators of all parts. ($a+b+c+d+f+g+h+k$)

10.

$E_1:\ 4+2\lvert 2-x\rvert=12$	$E_2:\ \lvert x+4\rvert+\lvert x+4\rvert=10$
$I_1:\ \lvert 2x+6\rvert<18$	$I_2:\ (x-5)^4<16$

A = the sum of the real solutions to equation E_1.
B = the least solution of the real solutions to equation E_2.
C = the number of integral solutions to inequality I_1. If there is an infinite number of integral solutions, then let C=100.
D = the number of integral solutions to inequality I_2. If there is an infinite number of integral solutions, then let D=100.

Give the value of A+B+C+D.

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11. A system of equations is given by $\begin{cases} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \\ a_3x+b_3y+c_3z=d_3 \end{cases}$, where all coefficients are integers.

Using Cramer's Rule, the x value of the solution $(x, y.z)$ is given by $\dfrac{\begin{vmatrix} 14 & -3 & 1 \\ -4 & 5 & -1 \\ 21 & -2 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 1 & 5 & -1 \\ 4 & -2 & 1 \end{vmatrix}}$.

Give the ordered triple $(x, y.z)$ of the solution to the system, in the form $(x, y.z)$.

---

12. For all parts below, $i=\sqrt{-1}$,

A = $\sum_{n=0}^{3}\left(\lvert 3+ni\rvert\right)^2$. B = $\sum_{n=0}^{99} i^n$

C= the sum of the non-real roots of $f(x)=x^3-3x^2+x+5$.
D = the product of the non-real roots of $f(x)=x^3-3x^2+x+5$.

Give the value of A+B+C+D.

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13. A = the solution over the real numbers of $2\log_3(x-1)-\log_3 2=1$.
B = the greatest solution over the real numbers of $\ln\left(x^2+10\right)=\ln 7+\ln x$.
C = the least solution over the real numbers of $\ln\left(x^2+10\right)=\ln 7+\ln x$.
D = the solution over the real numbers of $\log_5(x-1)-\log_5(x+1)=1-\log_5 7$.

Give the value of ABCD, written as $p+q\sqrt{r}$ in simplest form.

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14. $P_1:\ 8x^3+27$ $P_2:\ 64-y^3$ $P_3:\ 5z^4-80$
Factor all polynomials above completely so that factors have positive leading coefficients. Example: $8-2x^2$ should have factors $-2,(x-2),(x+2)$

A = the value of the binomial factor over integers of P_1 when $x=2$.
B = the value of the trinomial factor over integers of P_2 when $y=1$.
C = the sum of the factors of P_3 when $z=-1$. Note: P_3 is factored over integers, with three of the factors in the forms $a,\ bx+c,\ dx+h$.

Give the value of A+B+C.

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15. $x^2=25$, $y^2=36$, $z^2=100$.

A = the greatest possible value of $x+y+z$.
B = the least possible value of $x+y+z$.
C = the greatest possible value of $x-y$.
D = the least possible value of $x-y$.

Final answer is $2A+B+3C+D$.

	ANSWER	Intermediate Variables	Notes
1.	1784	A=0. B= -7. C= $-1792x^5y^3$. D= -1	
2.	$-14\sqrt{5}$	A = $\frac{-7}{8}$. B= 1. C= $\frac{1}{2}$. D= $2\sqrt{5}$	
3.	-1944	A= -7. B=9. C=9. D= $\frac{7}{24}$	
4.	$-\frac{4}{3}$	A= -4. $B=-1$. $C=\frac{1}{3}$. D= -1	Answer must be fraction form
5.	35450	A=1. B= $-\frac{102}{103}$. C= $\frac{1}{4}$. D=35552	
6.	178	A= -1. $B=\frac{1}{16}$. C = 2^{162}. D= 163.	
7.	616	A=220, B= $\frac{7}{10}$, D=4	There is no C because the C was used for combinations
8.	8	A=4, B=1, C=2, D=1	
9.	284	A= $\frac{4}{9}$, B = $\frac{5}{18}$, C= $\frac{40}{81}$, D= $\frac{1}{126}$	
10.	15	A=4, B= -9, C=17, D=3.	
11.	$(4,-1,3)$	x=4, y = -1, z=3	Ordered triple is needed
12.	59	A=50, B=0, C=4, D=5	
13.	$60+60\sqrt{6}$	A= $1+\sqrt{6}$. B=5. C=2. D=6	$p+q\sqrt{r}$ form needed
14.	36	A=7, B=21, C=8	
15.	43	A=21. B= -21. C=11. D= -11	

SOLUTIONS

1. Part A: $\frac{4-i}{5+3i}=\frac{(4-i)(5-3i)}{(5+3i)(5-3i)}=\frac{17-17i}{25+9}=\frac{1}{2}-\frac{1}{2}i=a+bi$. A = $(a+b)$ = 0

Part B: $(3-4i)(c+di)=(-33-56i)$. $c+di=\left(\frac{-33-56i}{3-4i}\right)\left(\frac{3+4i}{3+4i}\right)=\frac{125-300i}{25}=5-12i$

B = $(c+d)$.= -7.

Part C: The fourth term of $(2x-y)^8$ is $C(8,3)(2x)^5(-y)^3$ = $\frac{8(7)(6)}{3(2)(1)}\cdot 32x^5\cdot\left(-y^3\right)$ =

C = $-1792x^5y^3$

Part D: Let x=y=1 for $(x-2y)^9$ for a sum of the coefficients is -1.

Final value is A+B+C+D for $x=1$ and $y=-1$: 0+ $-7+1792-1=1784$

2. Part A, B: $x=2y^2-4y+1$. $x-1=2(y^2-2y)$. $x-1+2=2(y^2-2y+1)$. $(x+1)=2(y-1)^2$.

   For leading coefficient in standard form of 2 = $\frac{1}{1/2}$, the focus is $\frac{1}{2}$ divided by 4 = $\frac{1}{8}$ of a unit to the right from vertex $(-1,1)$ which puts focus at $\left(\frac{-7}{8},1\right)$ = (A, B).

   Part C: $x^2-4y^2-4x+32y-64=0$. $x^2-4x-4y^2+32y=64$.

   $x^2-4x+4-4(y^2-8y+16)=64+4-64$. $(x-2)^2-4(y-4)^2=4$. $\frac{(x-2)^2}{4}-\frac{(y-4)^2}{1}=1$

   which gives the slope of asymptotes $\pm\frac{1}{2}$ and C=$\frac{1}{2}$.

   Part D: Using the equation above in part C, foci will be twice $\sqrt{4+1}$ apart, so D=$2\sqrt{5}$.

   Final value is 16ABCD = $16\left(\frac{-7}{8}\right)(1)\left(\frac{1}{2}\right)\left(2\sqrt{5}\right)$ = $-14\sqrt{5}$

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3. Part A: $(x-1)^{\frac{4}{3}}-2(x-1)^{\frac{2}{3}}=8$. Let $a=(x-1)^{\frac{2}{3}}$. $a^2-2a-8=0$. $a=4$ or -2. Since $a=(x-1)^{\frac{2}{3}}$ cannot be negative due to the numerator of 2, we let $(x-1)^{\frac{2}{3}}=4$.

   $x-1=\pm 4^{3/2}$. $x=1+8,1-8=9,-7$. So A – the least integer solution which is -7.

   Part B: See part A and B=the greatest integer solution which is 9.

   Part C: $(x-1)^{\frac{4}{3}}=16$. . Since we want only the greatest solution, I will drop the negative option on the root, to get $x=1+16^{3/4}=9$.

   Part D: $2\left(\sqrt[3]{3x-1}\right)=-1$. $\sqrt[3]{3x-1}=-\frac{1}{2}$. $3x-1=-\frac{1}{8}$. $x=\frac{7}{24}$.

   Final value is $\frac{\text{ABC}}{\text{D}}$ = $\frac{-7(9)(9)}{7/24}=\frac{-7(81)(24)}{7}$ = -1944

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4. $f(x)=\frac{x^2-3x-4}{16-x^2}$ = $\frac{(x-4)(x+1)}{-1(x-4)(x+4)}$. There is a removable discontinuity at x=4.

   $g(x)=\frac{2}{x-3}-\frac{1}{3x+1}$ = $\frac{2(3x+1)-1(x-3)}{(x-3)(3x+1)}$ = $\frac{5x+5}{(x-3)(3x+1)}$

   Part A = the x-intercept of the vertical asymptote of f. The asymptote is $x=-4$. A= -4

   Part B =the x-intercept of the graph of f which is $B=-1$.

   Part C: $\frac{5x+5}{(x-3)(3x+1)}=-\frac{5}{4}$. Reduce the numerator by 5 and cross multiply.

   $-4(x+1)=(x-3)(3x+1)$. $3x^2-4x+1=0$. $(3x-1)(x-1)=0$. The least value is $\frac{1}{3}$

so C=$\frac{1}{3}$.

Part D: From $\frac{5x+5}{(x-3)(3x+1)}$ =0, we get $x=-1$.

Final answer is ABCD = $(-4)(-1)\left(\frac{1}{3}\right)(-1)=-\frac{4}{3}$

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5. Part A: $A=\sum_{n=1}^{1000}\frac{3}{4}\left(\frac{1}{4}\right)^{n-1}$ is the sum of a geometric series, and since the 100th term has a denominator of 4^{1000}, rounded to the whole number as requested, means that we can use the infinite sum formula and get the same answer.

$\frac{a_1}{1-r}=\frac{3/4}{1-1/4}=1$

Part B: the 100th term of $\frac{3}{4},-\frac{4}{5},\frac{5}{6},-\frac{6}{7},\ldots$. Note that this sequence is just $(-1)^{n+1}\frac{(n+2)}{(n+3)}$

so the 100th term should be $-\frac{102}{103}$.

Part C: The infinite sum of $2-4x+8x^2-16x^3+\ldots$ is equal to $\frac{2}{1-2x}$. Set this equal to $\frac{4}{3}$

to get $x=\frac{1}{4}$.

Part D: This is an arithmetic sequence. The sum is $\frac{n}{2}\left(2a_1+(n-1)d\right)$ =

$\frac{101}{2}\left(4+100(7)\right)=101\left(2+350\right)$. $101(352)=35552$.

Final answer is $412(ABC)+D$ = $412(1)\left(-\frac{102}{103}\right)\left(\frac{1}{4}\right)+35552=-102+35552$ = 35450

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6. Part A: $4\left(\frac{1}{8}\right)^{A+2}=2^A$. $2^{2-3(A+2)}=2^A$. $2-3A-6=A$. $A=-1$.

Part B: $\sqrt{1+\sqrt{B}}=\frac{\sqrt{5}}{2}$. $2\sqrt{1+\sqrt{B}}=\sqrt{5}$. $4(1+\sqrt{B})=5$. $\sqrt{B}=\frac{1}{4}$. $B=\frac{1}{16}$

Part D: $\log_2\left(\log_3\left(\log_4 C\right)\right)=2$. $\left(\log_3\left(\log_4 C\right)\right)=4$. $\left(\log_4 C\right)=81$. $C=4^{81}=2^{162}$. By the Fundamental Counting Principle, C will have factors all divisible by 2. So C's factors can be $2^0,2^1,2^3,\ldots,2^{162}$. D= the number of positive integral factors, so D=163.

Final answer is $A+D+\frac{1}{B}$ = $-1+163+16=178$

---

7. Part A: $C(12,3) = \frac{12!}{3!9!} = \frac{12(11)(10)}{3(2)(1)} = 220$. A=220.

   Part B: $P(9,3)/P(10,3)$ = 9(8)(7) divided by (10(9)(8) = $\frac{7}{10}$.

   Part C: $\frac{k!(k+1)!}{(k-1)!(k+2)!} = \frac{2}{3}$. $\frac{k\cancel{(k-1)(k-2)\ldots(1)}(\cancel{k+1)(k)(k-1)\ldots(1)}}{\cancel{(k-1)(k-2)\ldots(1)}\,(k+2)\cancel{(k+1)\ldots(1)}}$ = $\frac{k}{k+2}$. Now set $\frac{k}{k+2} = \frac{2}{3}$
   $3k = 2k+4$. k=4.

   Final answer is ABD = $220\left(\frac{7}{10}\right)(4) = 616$.

---

8. Part A: E_1 $x^2+(y-3)^2=25$ and E_3 $x^2+4y^2=16$. Solve for x^2, and substitute to get $25-(y-3)^2=16-4y^2$. $25-y^2+6y-9=16-4y^2$. $3y^2+6y=0$. $3y(y+2)=0$ so $y=0$ or $y=-2$. At y=0, we have points (4, 0) and (-4,0). At y=-2, we have x=0. So the greatest x-coordinate is 4, and A=4.

   Part B: E_2: $4x+5y=-1$ and E_5: $y=x-2$. $4x+5(x-2)=-1$. $9x=9$. $x=1$. B=1.

   Part C: the number of intersection points of E_3 and E_5 is 2. We have an ellipse with center at the origin, major axis endpoints at x= -4 and 4, and minor axis endpoints at y=2 and y= -2. The line has the y-intercept -2, so that is one point, and a slope of 1 will make it secant to the curve. C=2.

   Part D: The number of intersection points of E_5 and E_4 is 1. A quick look at slopes tells us that the two lines will meet and the maximum number of intersections of lines is 1.

   Final answer is A+B+C+D = 4+1+2+1 = 8.

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9. There are 5 blue, 4 red, and TOTAL 9 in the jar.

   A = one draw is red has probability $\frac{4}{9}$,

   B = $C(5,2)/C(9,2)$ = $\frac{5(4)}{2(1)} / \frac{9(8)}{2(1)} = \frac{20}{72} = \frac{5}{18}$.

   C = blue, red or red, blue. $\frac{5}{9}\left(\frac{4}{9}\right)+\frac{4}{9}\left(\frac{5}{9}\right)=\frac{40}{81}$

   D = $\frac{C(5,5)}{C(9,5)} = \frac{5(4)(3)(2)(1)}{9(8)(7)(6)(5)}$ = $\frac{1}{126}$

   For parts A, B, C and D written in reduced fraction form, your final answer is the sum of the numerators and denominators of all parts. 4+9+5+18+40+81+1+126 = 284

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10. Part A: $E_1:\ 4+|2-x|=12$. $2|2-x|=8$. $|2-x|=4$. $2-x=\pm 4$ gives $x=-2$ or 6. The sum is 4.

Part B: $E_2:\ |x+4|+|x+4|=10$. $2|x+4|=10$. $|x+4|=5$. $x+4=-5$ gives the least solution, which is -9.

Part C: $I_1:\ |2x+6|<18$ gives $-18<2x+6<18$. $-24<2x<12$. $-12<x<6$. Integers in the solution set are -11, -10, ... , 5. This gives -11 to -1 is 11, 1 to 5 gives 5, and 0 for a total of 17 integers.

Part D: $I_2:\ (x-5)^4<16$. All values will give a positive answer on the left of the inequality, so we need to find $|x-5|<2$. $-2<x-5<2$. $3<x<7$. Integers 4, 5, 6 give a number of integral solutions is 3.

Final answer is A+B+C+D = 4+ -9 + 17 + 3 = 15

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11. If the x solution corresponds to $\dfrac{\begin{vmatrix} 14 & -3 & 1 \\ -4 & 5 & -1 \\ 21 & -2 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -3 & 1 \\ 1 & 5 & -1 \\ 4 & -2 & 1 \end{vmatrix}}$ then the system is

$\begin{cases} 2x-3y+z=14 \\ x+5y-z=-4 \\ 4x-2y+z=21 \end{cases}$. Adding the first two gives $3x+2y=10$. Adding the last two gives $5x+3y=17$. Just for fun, subtracting the first and last, to get more options, gives $2x+y=7$. $(3x+2y=10)-2(2x+y=7)$ gives $-x=-4$ and x=4. y= -1 and $z=3$.

Options include doing the determinants using Cramer's Rule.

Final answer is $(4,-1,3)$

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12. A = $\sum_{n=0}^{3}|3+ni|^2$ = $|3|^2+|3+i|^2+|3+2i|^2+|3+3i|^2$ = $9+10+13+18=50$

B = $\sum_{n=0}^{99} i^n$ = $(i^0+i^1+i^2+i^3)+(i^4+i^5+i^6+i^7)+...+(i^{96}+i^{97}+i^{98}+i^{99})$ = 0

since each parentheses is equal to $(1+i-1-i)$. B=0.

C = the sum of the non-real roots of $f(x)=x^3-3x^2+x+5$. Since we can see that -1 is a real root, we divide and get x^2-4x+5 which has roots $2\pm i$. The sum of these last two roots is 4. C=4.

D = the product of the non-real roots of $f(x)=x^3-3x^2+x+5$. See part C and we get the product of $2\pm i$ which is 5. D=5.

Final answer is A+B+C+D = 50 + 0 + 4 + 5 = 59.

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13. A: $2\log_3(x-1)-\log_3 2=1$. $\log_3\dfrac{x^2-2x+1}{2}=1$. $\dfrac{x^2-2x+1}{2}=3$. $x^2-2x-5=0$.

$x=\dfrac{2\pm\sqrt{4+20}}{2}=1\pm\sqrt{6}$. Since $1-\sqrt{6}$ is not in the solution over the real numbers, A = $1+\sqrt{6}$.

B: $\ln\left(x^2+10\right)=\ln 7+\ln x$. $\ln\left(x^2+10\right)=\ln(7x)$. $x^2-7x+10=0$. x=2 or 5. B= the greatest solution which is 5.

C: See part B. The least solution is 2.

D: $\log_5(x-1)-\log_5(x+1)+\log_5 7=1$. $\log_5\dfrac{7x-7}{x+1}=1$. $\dfrac{7x-7}{x+1}=5$. $7x-7=5(x+1)$. $2x=12$ D=6.

Final solution is ABCD = $\left(1+\sqrt{6}\right)(5)(2)(6)=60+60\sqrt{6}$

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14. P_1: $8x^3+27 = \left(2x+3\right)\left(4x^2-6x+9\right)$ P_2: $64-y^3 = -1(y-4)(y^2+4y+16)$

P_3: $5z^4-80 = 5\left(z^4-16\right)=5\left(z-2\right)\left(z+2\right)\left(z^2+4\right)$

A: the binomial factors of P_1 when $x=2$ is (2x+3)= 7.

B: the trinomial factor of P_2 is when $y=1$ is $(y^2+4y+16)$= 1+4+16= 21.

C: the sum of the factors of P_3 when $z=-1$ gives $5+\left(z-2\right)+\left(z+2\right)+\left(z^2+4\right)$ = $5+-3+1+5=8$.

Final answer is A+B+C= 7+21+8 = 36

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15. x = 5 or -5. y= 6 or -6. Z= 10 or -10.

A = 5+6+10=21. B = -5-6-10= -21. C = $5-(-6)=11$. D = $-6-(5)=-11$

Final answer is $2A+B+3C+D$ = 42 + (-21) + 33 + (-11) = 21+22=43.