ALGEBRA TWO INDIVIDUAL TEST

March 2021 FAMAT Regional Competition

"NOTA", seen in choice E for each problem, means "None of the Above."

$i = \sqrt{-1}$.

Diagrams may not be drawn to scale.

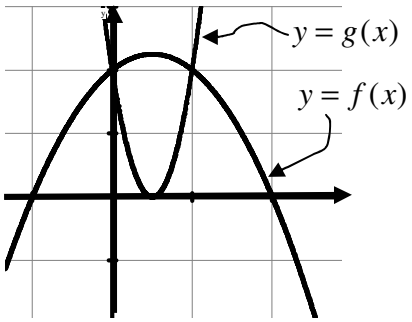
1. The inequality $5x^2 + 48x < 20$ has how many integer solutions?
   A. 12 B. 11
   C. 10 D. 9
   E. NOTA

2. The discriminant of $y = 2kx^2 - x - k + 1$ is 49. Which **can be** the $y-$intercept of the graph of $y = 2kx^2 - x - k + 1$?
   A. -5 B. -2
   C. 1 D. 4
   E. NOTA

3. The graphs of $x^2 - 2x + y^2 - 24 = 0$ and $y = 4$ intersect at two points. What is the distance between the two points?
   A. $3\sqrt{2}$ B. $4\sqrt{2}$
   C. 6 D. 8
   E. NOTA

4. For $i = \sqrt{-1}$, $(a + bi)(i - 2) = 10i$. Give the value of $a + b$.
   A. -6 B. -2
   C. 2 D. 6
   E. NOTA

5. For $|x + 2| + |2x + 4| = 4$, give the sum of the solutions, when solved over the real numbers.
   A. -4 B. -2
   C. 0 D. 2
   E. NOTA

6. 

The graph of $y = f(x)$ above has y-intercept 2, and roots -1 and 2. The graph of $y = g(x)$ above has y-intercept 2 and vertex on the x-axis at $x = \frac{1}{2}$. What is the positive distance between the y-coordinates of the points on the graphs of the two parabolas at $x = \frac{3}{4}$?
   A. $\frac{27}{16}$ B. $\frac{13}{12}$
   C. $\frac{31}{16}$ D. $\frac{7}{6}$
   E. NOTA

7. For the solution to $2\log(3x - 1) - \log(x + 2) = \log 28$ when solved over the real numbers, find the value of $\log(20x)$.
   A. 1 B. 1.5
   C. 2 D. 3
   E. NOTA

8. Solve: $2\left(4^{2n+1}\right) = \left(\frac{1}{2}\right)^n$ over the real numbers.
   A. -1 B. $-\frac{2}{5}$
   C. $-\frac{3}{5}$ D. $\frac{-2}{3}$
   E. NOTA

9. Where defined, $\frac{9-x^2}{2x^2-x-15}$ divided by $\frac{2x^2+9x+9}{2x^2-7x-15}$ gives a result of $\frac{a-x}{bx+c}$. Give the sum $(a+b+c)$.

A.12 B.8
C.7 D.5
E. NOTA

10. When solved over the real numbers, $6x^3-19x^2=4-16x$ has three solutions: $a,b,$ and c, for $a<b<c$. Give the value of $\frac{c}{b-a}$.

A. 6 B. 8
C. 10 D. 12
E. NOTA

11. In a pasture there are cows and egrets only. An egret is a white bird with two legs and a hearty "can do" attitude. There are 76 legs in the pasture and 24 heads. Assuming that all animals have the usual number of body parts, and that we have no other animals, insects, etc., and that there are no body parts lying around unattached to an animal, then how many wings are in the pasture?

A. 28 B. 26
C. 24 D. 20
E. NOTA

12. For the solution to $3\sqrt{1-4x}-\sqrt{2-x}=7$ over the real numbers, give the value of x^2-4x.

A. 12 B. 21
C. 45 D. 60
E. NOTA

13. Which of the following statement(s) is/are true about the simplest form of the expansion of $\left(2x^3-\frac{1}{x^2}\right)^{10}$?

I. There is a constant term.
II. There are 31 terms.
III. The least power is -20.

A. I, III only B. I, II only
C. II, III only D. II only
E. NOTA

14. The graph of $9x^2-16y^2=144$ has what distance between foci?

A. $2\sqrt{5}$ B. $2\sqrt{7}$
C. $2\sqrt{10}$ D. 10
E. NOTA

15. $\sqrt{380-\sqrt{380-\sqrt{380-\sqrt{...}}}}$ is closest to which value below?

A. 18 B. 19
C. 20 D. 21
E. NOTA

16. Given that $(\log_3 2)(\log_4(3x))=1.5$, then find the real value of x.

A. 3 B. 6
C. 9 D. 27
E. NOTA

17. $f(x)=\frac{1}{x-1}$ and $g(x)=\frac{2}{x}$. If $f(f(g(n)))=-\frac{1}{3}$, then find the value of n.
A. 6 B. 5
C. 4 D. 3
E. NOTA

18. The graph of $f(x)=x^2-4x+7$ has no real roots. The graph of $g(x)=f(x)+p$ has exactly one real root. Find the value of p.
A. -7 B. -4
C. -3 D. -2
E. NOTA

19. The probability of it raining in Tampa on Tuesday of next week is $\frac{1}{10}$. The probability of it raining in Tampa on Wednesday of next week is $\frac{2}{7}$. What are the odds that it will rain at least one of those two days in Tampa?
A. $5:14$ B. $5:9$
C. $27:70$ D. $27:43$
E. NOTA

20. The sixth term of an arithmetic sequence is 30 and the twenty-sixth term of the same sequence is 190. Give the sum of the first ten terms of this sequence.
A. 460 B. 420
C. 280 D. 260
E. NOTA

21. The graphs of $4(x-4)^2+25(y-2)^2=100$ and $2x-5y=8$ intersect at two points with coordinates (a,b) and (c,d). Find $2a+2c+3b+3d=$.
A. 43 B. 32
C. 28 D. 10
E. NOTA

22. $f(x)=4+2^{x-4}$ has inverse function $y=g(x)$. Give the value of $g(20)\bullet f(2)$.
A. 85 B. 68
C. 51 D. 34
E. NOTA

23. $f(x)=x^3-6$ has inverse relation $y=g(x)$. The two graphs intersect at point P on the $xy-$coordinate plane. Give the distance between the point P and the point with coordinates $(-3,-10)$.
A. 10 B. 12
C. 13 D. 15
E. NOTA

24. Find the sum of the series, for some terms (such $10\frac{1}{3}$) written as mixed numbers: $10+10\frac{1}{3}+10\frac{2}{3}+...+44$.
A. 2781 B. 2754
C. 2727 D. 2033
E. NOTA

25. A sample consists of five observations (five positive integers). The sample has a mean of 20 and a median of 24. The largest minus the smallest observations of the sample has what least possible value?
A. 3.4 B. 10
C. $12.\overline{3}$ D. 15
E. NOTA

26. There exist three positive integers R, S and T which are relatively prime. If $R\log_{400} 2 + S\log_{400} 5 = T$ then give the value of $R+S+T$.

A.6 B.7
C.8 D.9
E. NOTA

27. Line L_1 is parallel to $3x-y=6$ at the point $(0,-16)$. Line L_2 is perpendicular to $3x-y=6$ and passes through the x-intercept of $3x-y=6$. What is the intersection point of L_1 and L_2?

A. $(5,-1)$ B. $(2,4)$
C. $(6,-1)$ D. $(6,2)$
E. NOTA

28. The reflection of the graph of $5x-8y-14=0$ over the $x-$axis has equation written in $y=mx+b$ form, Give the value of $m+b$.

A. $-\frac{15}{56}$ B. $\frac{9}{8}$
C. $\frac{137}{40}$ D. $\frac{87}{40}$
E. NOTA

29. How many pairs of positive integers (r,s), such that $r+s\le 100$ will satisfy the equation $\frac{r^{-1}+s}{r+s^{-1}}=17$?

A. 8 B. 7
C. 6 D. 5
E. NOTA

30. For $a>0$, $\sqrt{a\sqrt{a\sqrt{a\sqrt{a}}}}=$

A. $a^{\frac{15}{16}}$ B. $a^{\frac{17}{8}}$
C. $a^{\frac{1}{4}}$ D. $a^{\frac{7}{8}}$
E. NOTA