February Regional **Pre-Calculus Individual Test**

For all of the following questions "NOTA" stands for "None of the above answers is correct". Good luck and have fun!

1.) Evaluate the $\sin(\frac{7\pi}{12})$.

A.) $\frac{\sqrt{6}+\sqrt{2}}{4}$ B.) $\frac{\sqrt{6}-\sqrt{2}}{4}$ C.) $\frac{\sqrt{3}+\sqrt{2}}{4}$ D.) $\frac{\sqrt{3}-\sqrt{2}}{4}$ E.) NOTA

2.) Simplify where defined: $\frac{cscx}{cotx} \cdot \frac{sinx}{tanx}$

A.) sinx B.) cosx C.) tanx D.) 1 E.) NOTA

3.) If the $\sin\alpha$ is equal to $\frac{3}{5}$ for $0 \leq a < \frac{\pi}{2}$ find
$(\cot\alpha + \csc\alpha) - (\tan\alpha + \sec\alpha)$

A.) 1 B.) -1 C.) $\frac{1}{6}$ D.) $\frac{-1}{6}$ E.) NOTA

4.) Find the sum of the solution(s) to the following equation over the interval $[0,2\pi)$
$\sin^2 x = \cos^2 x$

A.) 2π B.) 4π C.) 6π D.) 8π E.) NOTA

5.) Julian has a triangle with side lengths $\sqrt{19}$, 3 and 5. The second largest angle measure in the triangle can be written in the form $\frac{a\pi}{b}$ when fully reduced and where both a and b are relatively prime positive integers. What is the sum of $5a + b$?

A.) 5 B.) 6 C.) 8 D.)10 E.) NOTA

6.) Which of the following expressions is not equivalent to $\cos 2\theta$.

A.) $1 - 2\sin^2\theta$ B.)$2\cos^2\theta - 1$ C.)$\cos^2\theta - \sin^2\theta$ D.) $2\cos\theta\sin\theta$ E.) NOTA

7.) Mr. Otto and Mr. Snow are playing a game where both of them roll two different die and observe the outcome. Since Mr. Otto does not play fair, his die is rigged such that the probability of rolling a number k is kth times more likely than rolling the number 1. The die still is labeled from 1-6. Mr. Snow is using a fair six - sided die. The probability that Mr. Otto's roll is 2 or more than Mr. Snow's roll is equal to $\frac{\alpha}{\beta}$ when fully reduced and simplified. What is the sum of the digits of $\alpha + \beta$?

A.) 5 B) 9 C) 12 D) 16 E) NOTA

8.) How many solutions exist for $\tan^2\theta = 0$ over the interval $[0,2\pi)$?

A.) 0 B.) 2 C.) 4 D.) 6 E.) NOTA

9.) Find the sum of the solution(s) to the following equation with the domain $0 \leq x < 2\pi$.

$$\cos^2 x - 3\sin x = -3$$

A.) $\frac{\pi}{2}$ B.) π C.) 2π D.) 3π E.) NOTA

10.) If the $\cos\theta = \frac{12}{13}$ where $\pi \leq \theta < 2\pi$ then the $\sin 2\theta$ can be written in the form $\frac{a}{b}$ when fully reduced and where $a < b$. Find the units digit of $a + b$.

A.) 3 B.) 5 C.) 7 D.) 9 E.) NOTA

11.) The sum of the solution(s) of the following equation over the interval $[0,2\pi)$ can be written in the form $\frac{a\pi}{b}$ when fully reduced. Find $a + b$.

$$2\sin^2 x - \sin x - 1 = 0$$

A.) 3 B.) 5 C.) 7 D.) 9 E.) NOTA

12.) Triangle ABC has side lengths a, b and c. It has angles A, B and C. Angle A is opposite of side a. Angle B is opposite of side b. Angle C is opposite of side c. Find b if angle B is $120°$, angle C is $30°$and c has a length of 2.

A.) $\sqrt{3}$ B.) $2\sqrt{3}$ C.) $3\sqrt{3}$ D.) $2\sqrt{2}$ E.) NOTA

13.) Stanley has a triangle with side lengths of 2, 4, and 4. What is its area?

A.) $\sqrt{5}$ B.) $\sqrt{15}$ C.) $2\sqrt{5}$ D.) $2\sqrt{15}$ E.) NOTA

14.) Sara is trying to improve her geometry skills. She is trying to find the area of the triangle with the following vertices on the Cartesian coordinate plane with coordinates (2,3) (0,5) and (1,4). What is the area of Sara's triangle?

A.) 2 B.) 1 C.) $\frac{1}{2}$ D.) $\frac{1}{4}$ E.) NOTA

15.) Congratulations!!! You have made it halfway through this test. Here is a nice easy question to help soothe your brain. Only one of the following statements are true. The answer to this question is the answer choice that has the true statement.

A.) B is false B.) C is false C.) D is true D.) E is true E.) NOTA

16.) It takes Emily ten hours to build a couch from Ikea. It takes Khushi five hours to build the same couch from Ikea. Jayita takes the same amount of time to build the couch as Khushi. How long will it take for all three of them to build this couch from Ikea if they work together? Round to the nearest hour. Each person works at their respective constant individual rates (given above), when working as a team.

A.) 1 hour B.) 2 hours C.) 3 hours D.) 4 hours E.) NOTA

17.) The square of the distance between polar points $(5, \frac{\pi}{3})$ and $(2, -\frac{\pi}{4})$ can be written in the form $a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6}$. Find the sum of the digits of $4a + 3b + 2c + d$.

A.) 4 B.) 7 C.) 9 D.) 10 E) NOTA

18.) Evaluate: $(i^{1010} \cdot i^{2020} \cdot i^{3030} \cdot i^{4040}) - (i^{1010} + i^{2020} + i^{3030} + i^{4040})$

A.) i B.) $-i$ C.) 1 D.) -1 E.) NOTA

19.) The relation $x^2 + y^2 = 1$ is considered to be:

A.) Odd B.) Even C.) Both D.) Neither E.) NOTA

20.) Duncan and Nahim are arguing over how to identify a non-degenerate conic by its eccentricity. Duncan states that if the eccentricity of a conic is 1, then it is a hyperbola. However, Nahim states that if the eccentricity of a conic is 1, then it is a parabola. Who is correct?

A.) Duncan B.) Nahim C.) Both D.) Neither E.) NOTA

21.) The solution to the following expression can be written in the from $\alpha + \beta\pi e$ where α and β are relatively prime positive integers. Find $\alpha + b$. $\begin{vmatrix} 7 & e \\ \pi & 4 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix}^{999}$

A.) -29 B.) -27 C.) 27 D.) 29 E.) NOTA

22.) Solve over the real numbers: $\log_2(x + 2) + \log_2(5) = 4$

A.) $\frac{6}{5}$ B.) $\frac{-6}{5}$ C.) -3 D.) 9 E.) NOTA

23.) Simplify the expression where $x \neq 0$: $\frac{6x^{-2} + 9x^2}{3x^{-2}}$

A.) $2x^4 + 3$ B.) $9x^2 + 2$ C.) $3x^4 + 2$ D.) $\frac{1}{2}x^2 + 9$ E.) NOTA

24.) Solve for k: $2^{2k-1} = 8^{k+5}$

A.) -6 B.) 2 C.) -2 D.) -16 E.) NOTA

25.) Let the cube of the distance between the centers of the following conic sections be equal to A. What is the tens digit of A?

Conic 1: $2x^2 + 2y^2 - 8x + 4y + 2 = 0$
Conic 2: $x^2 - 4y^2 + 4x + 16y - 21 = 0$

A.) 0 B.) 1 C.) 4 D.) 6 E.) NOTA

26.) The maximum point on the parabola $y = -2(x-m)(x-n)$ (where m and n are real numbers) can be written in the form (x,y). The x-coordinate can be expressed in terms of m and n. Find this expression.

A.) $m + n$ B.) $\frac{m+n}{2}$ C.) $m^2 + n^2$ D.) $\frac{m^2+n^2}{2}$ E.) NOTA

27.) Ms. Lambert has Matrix A whose determinant is T. In terms of T find the determinant of the inverse of the transpose of Matrix A. Matrix A is a square and invertible matrix.

A.) $-T$ B.) $-\frac{1}{T}$ C.) T D.) $\frac{1}{T}$ E.) NOTA

28.) Given the following: $|\vec{a} \times \vec{b}| = 12$, $\vec{a} \cdot \vec{b} = 6$, $\vec{a} = <1,-1,1>$, and $\vec{b} = <2,x,y>$ When solved for x and y, $x = \delta \pm \alpha\sqrt{\beta}$ and $y = \varepsilon \pm \alpha\sqrt{\beta}$. What is $\alpha + \beta + \delta + \varepsilon$?

A.) 8 B.) 10 C.) 12 D.) 16 E.) NOTA

29.) How many real solutions exist to the following equation?

$$|2x + 1| + |x - 3| = 9$$

A.) 0 B.) 1 C.) 2 D.) 3 E.) NOTA

30.) Which of the following are true?

I. All parabolic functions have 2 distinct roots.

II. Assume that when x approaches ∞ then f(x) also approaches ∞. Then, we can conclude that when x approaches $-\infty$ then f(x) also must approach $-\infty$.

III. A complex number multiplied by its conjugate is always real.

A.) III only B.) I and II C.) I and III D.) II and III E.) NOTA