TEAM ANSWERS

Part A	Part B	Part C	Part D	
1. 114	$\frac{24}{11}$	$\frac{37}{2}$	87	Parts B and C must be fraction form.
2. 21	16	1	31	
3. 15	25:9 or $\frac{25}{9}$	64π	108	Part B asks for the ratio.
4. $216\sqrt{3}$	$6\sqrt{3}$	$45\sqrt{3}$	$12\sqrt{3}$	
5. 30	15840	720	24	
6. $\frac{65}{17}$	$\frac{91}{34}$	$\frac{27}{5}$ or 5.4	$\frac{4}{3}$	Part D must be fraction form.
7. 288	96	$72\sqrt{3}$	$\frac{2}{3}\sqrt{3}$	
8. 60	25	2π	acute, scalene	Order is not important. Spelling is important.
9. 3	30	12	yes	
10. $9\sqrt{3}$	$21\sqrt{3}$	30	400	
11. $100\sqrt{2}$	$75\sqrt{3}$	$150\sqrt{3}$	25π	
12. 300	π	$3\sqrt{13}$	$2\sqrt{10}$	
13. 8	105	$\frac{1}{2}$ or 0.5	$\frac{3+\sqrt{3}}{4}$	Part D must be in form $\frac{r+\sqrt{s}}{t}$ $t>0$
14. $\frac{69}{2}$ or 34.5	30	204	47.5 or $\frac{95}{2}$.	
15. 20	4	20	60	

TEAM SOLUTIONS

1. Part A: $(36-2x)+(6x+24)+(4x)=180$. $8x=120$. $x=15$. The angles have measures 6, 114 and 60 degrees. The largest measure is 114.

   Part B: $m\angle R+m\angle A+m\angle T=180$. $3(6x+24)+36-2x+6x+24=180$. $22x+132=180$. $x=\frac{24}{11}$.

   Part C: case 1: $\angle$H is 90 degrees. $6x+24+36-2x=90$. x=7.5. In this case, all angles have valid measures. Case 2: $6x+24=90$. x=11. The acute angles will then be 14 and 76. Case 3: $36-2x=90$. x= -27. That gives a negative measured angle. So we discard this value. Answer = 7.5 + 11 = $\frac{37}{2}$.

   Part D: If the longest side is $\overline{SA}$ then the obtuse angle must be angle T. $90<6x+24<180$. $11<x<26$. $0<(36-2x)<90$. $-27<x<18$. The intersection of these values (because both situations must happen at the same time) gives integers 12, 13, 14, 15, 16, 17. Checking the least and greatest of these values gives x=12 for angles 12, 96 and 72. The largest x is 17 for angle measures 2, 126 and 52. Both work. The sum of possible values of x is 87.

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2. Part A: $6x-8+3x-1=180$. x=21.

   Part B: Using part A, $m\angle UST=118$ so $m\angle RSU=62$. So $4y-2=62$. y=16.

   Part C**:** $m\angle$RSV=118 so $m\angle YSV=59$. $m\angle YVS=180-59-60=61$, $m\angle YVW$ =62-61=1.

   Part D: VY is the shortest side length, because it is opposite the least angle measure. So SV and SY are each greater than 10. We know that SV+SY is greater than 10 so together we have the perimeter is greater than 30. Note that each of the two unknown side lengths could be 10.1 each. Answer=31.

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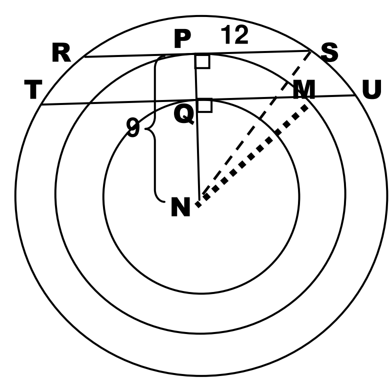
3. Part A: Consider right triangle PNS. Legs are 9 and 12, and NS completes the 3(3-4-5) triple with 15.

   Part B: If PN=9, which is the radius of the middle size circle, and the radius of the largest circle is 15 (see part A) then the ratio of radii is 15:9 = 5:3. And the ratio of the areas is 25:9.

   Part C: The area outside of the smallest circle and inside of the middle circle is $\pi(NM)^2-\pi(NQ)^2$. We know that $(NM)^2-(NQ)^2=(QM)^2$ by the Pythagorean Theorem, using triangle QMN. QM=8 so $(NM)^2-(NQ)^2=64$. The answer then is 64π

   Part D: Area of ΔRNS is $\frac{1}{2}(24)(9)=108$



4. Part A: Since one side of the hexagon is 12, the area is $\frac{3}{2}side^2\sqrt{3} = \frac{3}{2}(144)\sqrt{3} = 216\sqrt{3}$.

Part B: Angle K is 120 degrees so we have an isosceles triangle AKD with legs 6 and vertex angle 120 degrees. Drop a height from K and we get height is 3. Base is twice the long leg of the 30-60-90 triangle with hypotenuse AK. Base = AD = $6\sqrt{3}$.

Part C: From part B, we know $m\angle KAD = 30^o$, and since angle DAB is 90 we have angle EAB is 60 degrees. Drop the height of trapezoid EABF from E, and we have EA=6 so the height is $3\sqrt{3}$. The short leg of the 30-60-90 triangle with hypotenuse AE is 3 so AB=18. Area of the trapezoid is $\frac{1}{2}(3\sqrt{3})(12+18) = 45\sqrt{3}$.

Part D: We already found the distance from A to B in part C above, and it is the same as the distance from D to C. We found the distance AD is part A, Using the Pythagorean Theorem, we get $\left(6\sqrt{3}\right)^2 + (18)^2 = (BD)^2$. $6^2\left(\sqrt{3}\right)^2 + (3)^2 = (BD)^2$

BD = $\sqrt{6^2(12)}$ = $12\sqrt{3}$

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5. Part A: $180(n-2) = 1800$. $n-2 = 10$. n=12. One exterior angle is 360/12 = 30.

Part B: 360/4 = 90 sides. Interior sum is $(90-2)180 = 16200 - 360 = 15840$.

Part C: One exterior angle is 0.5 degrees, and 360/0.5= 720 sides.

Part D: $\frac{1}{2}n(n-3) = 252$. Trial and error is a great idea, or you can solve the quadratic $n^2 - 3n - 504 = 0$. $(n-24)(n+21) = 0$. n=24 or -21. So the answer is 24.

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6. Part A: Let EJ=x. Since $\overline{FJ}$ is an angle bisector of $\angle EFG$, $\frac{x}{5} = \frac{13-x}{12}$. $12x = 65 - 5x$.

$EJ = \frac{65}{17}$.

Part B: Since M is the midpoint of $\overline{EG}$, it has length $\frac{13}{2}$. So JM = $\frac{13}{2} - \frac{65}{17}$ = $\frac{91}{34}$.

Part C: Using Geometric Mean ratios, and XY=15, we get $9^2 = 15(XV)$ and XV= $\frac{27}{5}$ or 5.4

Part D: Since $\Delta VYZ \sim \Delta YXZ$, $\tan(\angle VYZ)$ = $\tan(\angle YXZ)$ = $\frac{12}{9} = \frac{4}{3}$

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7. Part A: The diameter of the circle is the diagonal of the square. So diagonal is 24. The side of the square is $12\sqrt{2}$. The area is this value squared, or 288.

Part B: The hexagon's side is the same length as the radius of the circle. So the circle has radius 12. A square is circumscribed about a circle of radius 12. Its side has length 24. So its perimeter is 24(4)= 96.

Part C: Triangle PQR has three sides which are all radii of the congruent circles. The area of this triangle is $\frac{side^2}{4}\sqrt{3} = 36\sqrt{3}$. Quadrilateral PRQS consists of two of these triangles, which will give area $72\sqrt{3}$

Part D: Let the square have perimeter 4 and side 1. That means the hexagon has perimeter 4 and side $\frac{4}{6}=\frac{2}{3}$. That makes area of the square 1 and area of the hexagon is $\frac{3}{2}side^2\sqrt{3}=\frac{3}{2}\left(\frac{2}{3}\right)^2\sqrt{3} = \frac{2}{3}\sqrt{3}$. The ratio is $\frac{2}{3}\sqrt{3}$.

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8. Part A: $80=\frac{1}{2}(100+RS)$. Arc RS has measure 60 degrees.

Part B: $x+10=\frac{1}{2}(4x+70-100)$. $2x+20=4x-30$. x=25.

Part C: Using parts A and B above, we have arcs 100, 170, and 60 degrees. Subtract the total of these from 360 and we get degree measure 30 degrees. Arc length for $\widehat{SQ}$ is $\frac{30}{360}(24\pi) = 2\pi$.

Part D: Since x=25 (from part B) we have angles 80, 35 and 65. Therefore the triangle is acute and scalene.

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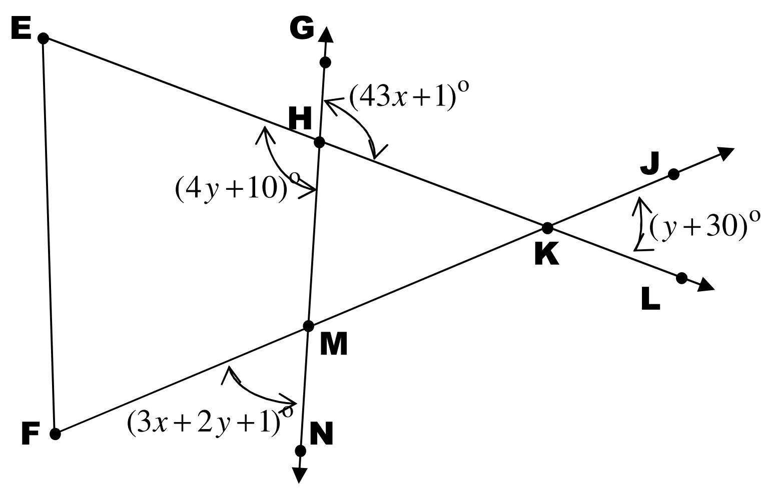
9. Parts A and B: $43x+1=4y+10$ and $y+30+3x+2y+1+180-(4y+10)=180$.
So $43x-4y=9$ and $3x-y=-21$.
-4 times the 2nd equation added to the first gives $31x=93$.



Part A: x=3

Part B: $3(3)-y=-21$. y=30.

Part C: If we fill in all of the angles for ΔHKM, we get angles K, M and H of that triangle are 60, 70 an 50 respectively. So angles E and F, of ΔKHM must add to 120.
$10a+(2a+12b-24)=120$.
12a+12b=144. So a+b=12.

Part D: Yes. Corresponding angles cannot be congruent, but ΔHKM can be similar to the correspondence in ΔFKE. Answer = yes.

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10. Part A: Since the area of the trapezoid is ¼ of the area of ΔRTS, then the area of ΔRUN is ¾ of the area of ΔRTS. So the ratio of the areas of ΔRUN to ΔRTS is ¾ : 1 or 3:4. The ratio of the sides of these similar triangles is $\sqrt{3}$:2. That means $\frac{\sqrt{3}}{2}=\frac{UN}{18}$. UN = $9\sqrt{3}$.

Part B: By the same reasoning as part A, $\frac{\sqrt{3}}{2} = \frac{perim}{60}$ for perim = the perimeter of triangle RUN. So the perimeter of ΔRUN is $30\sqrt{3}$. Using part A, we have UN=$9\sqrt{3}$. So RU+RN= $30\sqrt{3} - 9\sqrt{3} = 21\sqrt{3}$.

Part C: The area of triangle RUN is 90. This is 3/4 of the area of triangle RTS. So $90\left(\frac{4}{3}\right)$ = 120 for the area of triangle RTS. Subtract 120-90 to get 30.

Part D: We were told that NUTS has area ¼ of that of ΔRTS. So the area of ΔRTS is 400 square inches.

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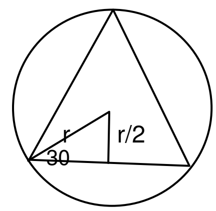
11. Part A: The diameter of the circle is the side length of the square. $100 = \pi d$. $d = \frac{100}{\pi}$.

The diagonal of the square will then be $\frac{100}{\pi}\sqrt{2}$. $\frac{A}{\pi} = \frac{100}{\pi}\sqrt{2}$ so A = $100\sqrt{2}$.

Part B: The inscribed circle will have radius the same as circle 2, and the apothem of the triangle has half the length. The side of the triangle is $r\sqrt{3}$. Area is $\frac{side^2}{4}\sqrt{3}$ = $\frac{\left(3r^2\right)\sqrt{3}}{4}$.



The radius of circle 2 is $\pi r^2 = 100$. $r = \sqrt{\frac{100}{\pi}}$. So area of the triangle is $\frac{3}{4}\left(\frac{100}{\pi}\right)\sqrt{3}$ = $75\sqrt{3}$.

Part C: The area of a regular hexagon is $\frac{3}{2}side^2\sqrt{3}$. The radius of the circle is the same as the radius of the hexagon, and the same as the side of the hexagon. $\frac{3}{2}\left(\sqrt{\frac{100}{\pi}}\right)\sqrt{3}$. Hexagon area is $\frac{150\sqrt{3}}{\pi}$. C = $150\sqrt{3}$

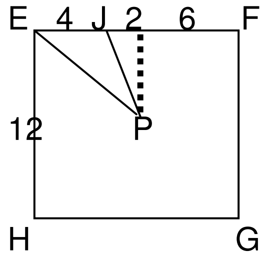
Part D: The radius of circle 1 is $\frac{50}{\pi}$ and so area of circle 1 is $\frac{2500}{\pi}$. The ratio of the areas is $\frac{2500}{\pi}$: 100 = $\frac{25}{\pi}:1$ = $25:\pi$ = $25\pi:\pi^2$. That makes D = 25π.

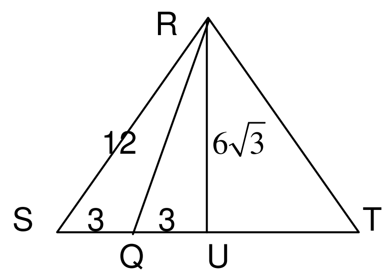
12. Part A: The dimensions of R_1 are 4 and 3. In R_2, the sides are 4k and 3k.
$16k^2+9k^2=625$. $25k^2=625$. k=5. So the area of R_2 is 20(15)=300.

Part B: The ratio of circumference to diameter is always pi. Answer = π.

Part C: Using ΔRQU, $3^2+(6\sqrt{3})^2=C^2$.
$C=\sqrt{9+36(3)}=3\sqrt{1+12}=3\sqrt{13}$.

Part D:





EJ=4 and J to the midpoint of EF is 2. $JP=\sqrt{4+36}=2\sqrt{10}$.

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13. Part A: The side of the equilateral triangle is $\sqrt{1+\sqrt{3}}$. Drop the height from S. That is the long leg of a 30-60-90 triangle so it is $\frac{1}{2}\left(\sqrt{1+\sqrt{3}}\right)\sqrt{3}$ = $\frac{1}{2}\sqrt{\sqrt{3}+3}$. $\frac{\sqrt{a+\sqrt{b}}}{c}$ form gives $a=3, b=3, c=2$. $a+b+c=8$

Part B: m$\angle PRQ=45=y+z$ (see diagram). m$\angle PRN=60=x+y$. x+y+z=90. 60+45= 105=x+2y+z. Subtract 90 to get y=15. So now in triangle STR we have 60+15 degrees which leaves 105 for $m\angle STR$. Answer = 105.

Part C: Drop the height from T to side $\overline{PN}$. In ΔPTU shown, we have 45 degrees at P and 30 degrees at N. Let UT=h. UN= $h\sqrt{3}$ and PU=h. So $PN=h+h\sqrt{3}$ = $h(1+\sqrt{3})=\sqrt{1+\sqrt{3}}$. Square both sides. $h^2(1+\sqrt{3})^2=(1+\sqrt{3})$. Divide to get $h^2=\frac{1}{1+\sqrt{3}}$. Area of ΔPTN is $\frac{1}{2}\left(\sqrt{1+\sqrt{3}}\right)\left(\frac{1}{\sqrt{1+\sqrt{3}}}\right)=\frac{1}{2}$.

Part D: Area of an equilateral triangle is $\frac{1}{4}side^2\sqrt{3}$ = $\frac{1}{4}\left(1+\sqrt{3}\right)\sqrt{3}=\frac{3+\sqrt{3}}{4}$

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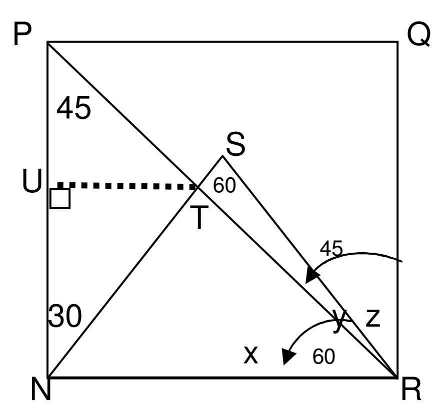

14. Part A: Vertex angle is R, so $2a+18+2(a+12)=180$. $a=\frac{138}{4}=\frac{69}{2}=34.5$

Part B: Vertex angle is F, so $4(m\angle G)+2(m\angle G)=180$. $m\angle G=30^{\circ}$ So $m\angle E=30^{\circ}$.

Part C: Case 1: $20+20+c+12=180$. c=128. Case 2: $20+2(c+12)=180$. c=68.
Case 3: $c+12=20$. c=8. Sum = 128+68+8 = 204.

Part D: Case 1: $2d-4=3d+6$. d= -10. But that gives a negative degree angle.
Case 2: $2d-4+6d+12=180$. d=172/8=21.5. That gives angles of 39, 70.5 and 70.5. This one works.
Case 3: $4d-8+3d+6=180$. d=26. Angles 84, 48 and 48. This one works.
Sum = 21.5+26 = 47.5 or $\frac{95}{2}$.

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15. Part A: Area of a rhombus is found by half the product of the diagonals.
$\frac{1}{2}\left(\frac{1}{2}x-2\right)\left(\frac{1}{2}x-4\right)=x+4$. Multiply by 8: $(x-4)(x-8)=8x+32$. $x^2-12x+32=8x+32$.
$x^2-20x=0$. x=0 or 20. If x=0, we have negative length diagonals, so let x=20.
To check: diagonals will be 8 and 6, for area 24. Answer = 20.

Part B: $(y+9)^2=9y^2+25$. $8y^2-18y-56=0$. Reduce: $4y^2-9y-28=0$. By the way, when you see the width 5, you might suspect a 5-12-13 triple, which does work for y=4. Back to the quadratic. $(y-4)(4y+7)=0$. y=4.

Part C: Diagonals of the rhombus are 6 and 8 so diagonals create four 3-4-5 triangles and each rhombus side is 5. The perimeter is 20.

Part D: From part B, we have side lengths are 5 and 12 for area 60.
