# GEOMETRY INDIVIDUAL TEST

*February Regional Competition*

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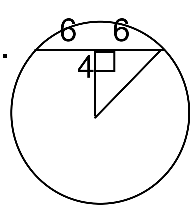
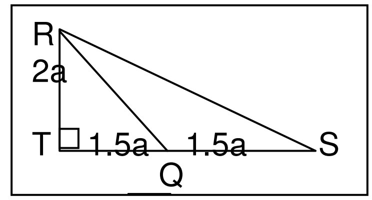
|---|---|---|---|---|
| 1. B | 7. D | 13. B | 19. D | 25. B |
| 2. C | 8. B | 14. D | 20. D | 26. C |
| 3. C | 9. A | 15. B | 21. B | 27. D |
| 4. A | 10. B | 16. C | 22. C | 28. A |
| 5. B | 11. D | 17. C | 23. D | 29. C |
| 6. C | 12. A | 18. A | 24. A | 30. D |

**Solutions:**

1. **B**. $\frac{120}{360}(18) =$ minor arc length=6. Major arc length is 18-6=12.
2. **C.** Perimeter=600 so side=100. The radius of the hexagon, the apothem and half of a side make a 30-60-90. The apothem has length $50\sqrt{3}$.
3. **C.** Since $\overleftrightarrow{RS}$ and $\overleftrightarrow{TU}$ intersect at point X., all points on those lines are coplanar. $\overleftrightarrow{XY}$ is perpendicular to $\overleftrightarrow{RS}$. If Y is not in the same plane then it will also be perpendicular to $\overleftrightarrow{RS}$. So I is true. II is true only if X is between T and U, which may not be true. III must be true since Y is not in the same plane as $\overleftrightarrow{RS}$, the definition of skew lines is that they are noncoplanar lines. I and III are true.
4. **A.** Using vertical angles MNL and PNQ we have $3x-4y = x+y-5$ so $2x-5y=-5$. Using the linear pair $\angle MNL$ and $\angle LNQ$ we get $5x-5y=175$ or $x-y=35$. $2(35+y)-5y=-5$. $-3y=-75$ $y=25$. x=60. x+y=85.
5. **B.** See diagram to the right. radius = $\sqrt{36+16} = 2\sqrt{13}$. Area is $\pi r^2 = 52\pi$




6. **C.** Using the two smallest sides, $\sqrt{6^2+8^2} = 10$ would make RT a length to make angle S a right angle. Since RT > 10, $m\angle S > 90^{\circ}$. The triangle is obtuse.
7. **D.** Let RT=2a and TS=3a as $Tan(\angle SRT) = \frac{3}{2}$.




Since Q is the midpoint ($\overline{RQ}$ is a median)., TQ=1.5a. RQ=$\sqrt{4a^2+2.25a^2}$ = $\frac{a\sqrt{16+9}}{2} = \frac{5}{2}a$. $\sin(\angle QRT) = \frac{1.5}{2.5} = \frac{3}{5}$

The length of $\overline{RS}$ is not needed.

8. **B .** Since the lines are parallel, $80+x = y+70$. $y-x=10$. Square both sides to get $x^2+y^2-2xy=100$. Substitute $xy=56$ to get $x^2+y^2=212$
9. **A.** Diagonals are perpendicular so two diagonals will create 4 congruent right triangles with leg 6 and hypotenuse 10. Use the Pythagorean Th. to get let 8 and diagonal 16. $A = \frac{1}{2}d_1d_2$ = $\frac{1}{2}(12)(16) = 96$.
10. **B.** Since RS=ST, vertex angle is S. So $(2x+10)+2(x-5)=180$. $4x=180$. x=45.
11. **D.** If one interior angle is $175^{\circ}$, then one exterior angle is $5^{\circ}$. So the polygon has $360/5=72$ sides. The 2nd polygon will have 8 sides, and 360/8=45 degrees.
12. **A.** Diagonals = $\frac{1}{2}n(n-3) = n+3$. $n(n-3) = 2n+6$. $n^2-5n-6=0$. $(n-6)(n+1)=0$ gives n=6. The interior of this polygon has $180(6-2)$ degrees = 720.
13. **B.** Let Q be at (0, y). $\frac{0-y}{10-0} = -4$. y=40. The distance PQ = $\sqrt{(10-0)^2+(0-40)^2} = 10\sqrt{17}$

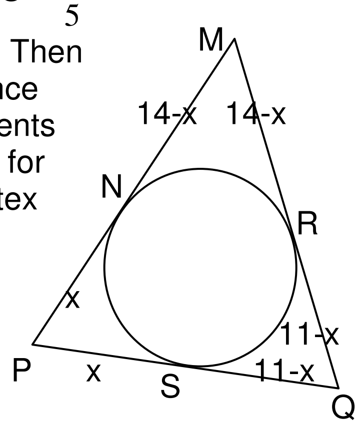
14. **D.** $(100p-70)+(120p-130)$ +100+ $34(p+1)+(30p+38)=540$. $284p=568$. p=2. The area of an equilateral triangle with side p is $\frac{p^2}{4}\sqrt{3}$ = $\sqrt{3}$.

15. **B.** Drop the height from Q to make side SR divided into 12 and 6. The height is then 8 by the Pythagorean Th. So area of the trapezoid is $\frac{1}{2}(8)(12+18)=120$

16. **C.** $80+20c+10+60+10c=360$ c= 70. So $m\angle R=\frac{1}{2}(150-70)$ =40

17. **C.** If legs of $\Delta RST$ 9 and 12, ST=15 to complete the Pythagorean Triple. Using a geometric mean formula, $9^2=15(VS)$. VS = $\frac{27}{5}$

18. **A.** Let PN=x. Then MN=14-x.. Since tangent segments are congruent for and angle vertex outside of the circle, PS=x and SQ=11-x. Fill in MR=14-x and RQ=11-x. That makes $(14-x)+(11-x)=13$. $25-2x=13$. x = 6.




19. **D.** If R is the largest angle then ST is the longest length for a side. So RT must be less than 120. Using the Triangle Inequality Theorem, RT must be between 20 and 220. So $20<RT<120$. That means RT can be 21 through 119, inclusive. 99 possible integers.

20. **D.** $\pi(r+2)^2-\pi r^2=32\pi$. $4r+4=32$ $r+1=8$. r=7 for the smaller circle.. Circumference total is $14\pi+18\pi=32\pi$.

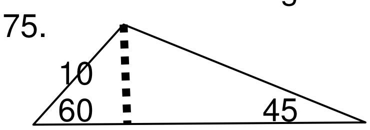
21. **B**. The right angle is either $(3x-12)^{o}$ or $(x+10)^{o}$ or neither. In the first case, x=34, and the other angles are 44 and 46 degrees. In the 2nd case, x+10=90 and x=80 and the other angles are 228 and nothing. So we do not use this case. In the 3rd case, 3x-12+x+10=90 gives x=23, and the angles are 57, 33 and 90. So possible values of x are 34 and 23 for a sum of 57.

22 .**C.** If the perimeter is 2k then each side is k/4. RSTU is a quadrilateral with two angles 135 degrees, and RS=UV so we have angles SRZ=ZVU=45. So now angle YRZ is 135-45=90 and similarly angle ZUV, TSV, etc. The shaded triangles are congruent, and 45-45-90 triangles. Combined area is $\frac{1}{2}\left(\frac{k}{4}\right)\left(\frac{k}{4}\right)$ doubled, which is $k^2/16$. So $\frac{k^2}{16}=k$. Since k is not zero, k=16.

23. **D.** AD= $\sqrt{5}$. $m\angle RAB+m\angle TAD=90$ so the angles of $\Delta RAB$ and $\Delta TAD$ are $x, 90-x, 90$ degrees and by AA are similar. RB=8 and BU=2. AB= $4\sqrt{5}$ and the shaded area is the large rectangle area minus the two triangles and the small rectangle area.

$$10(6)-\frac{1}{2}(1)(2)-\frac{1}{2}(4)(8)-\left(\sqrt{5}\right)\left(4\sqrt{5}\right)=$$
$$60-1-16-20=23$$

24. **A.** $3a+4a+5a=180$. $a=15$ and angles are 45, 60 and 75. See diagram. Altitude to the longest side has length $5\sqrt{3}$, and the longest side has length $5\sqrt{3}+5$



25. **B.** Since the small and large triangles are similar, but $\overline{ST}$ is NOT parallel to $\overline{PR}$, $\angle QST$ and $\angle QPR$ are not congruent (contrapositive of one of the corresponding angle theorems) so $\angle QST$ must be congruent to $\angle PRQ$ (angles of similar triangles are congruent, and angle Q is reflexive). So $\Delta QST \sim \Delta QRP$. $\frac{8}{PR} = \frac{8.5}{10}$. PR=$\frac{160}{17}$

26. **C**. R and S are consecutive angles so $4x + 40 + 2x + 20 = 180$. x=20. That gives R 120 degrees and T must have the same. So y=50. $y - x$ = 30.

27. **D.** Using $\Delta GHK \sim \Delta NJK$ we have $\frac{6}{8} = \frac{NJ}{10}$. NJ=$\frac{15}{2}$. Using $\Delta JML \sim \Delta HGK$ we have $\frac{MJ}{10} = \frac{7}{8}$. MJ= $\frac{35}{4}$. Subtract to get 5/4 for NM.

28. **A.** Since T is at x=5 then the circle has radius 5 and diameter 10. Since Q is a right angle, VP is a diameter also, being arc VTP must be 180 degrees. So VP=10.

29. **C.** If you use the large circle and secant and tangent segments, we get $(ST)^2 = 10(15)$ and $ST = \sqrt{150}$. Now use the small circle and secant and tangent segments:

$\left(\sqrt{150}\right)^2 = SR(SR + 5)$. Let SR=x.

$x^2 + 5x - 150 = 0$. $(x + 15)(x - 10) = 0$.
x=SR=10.

30. **D.** Drop the height to $\overline{TS}$ to give you a 30-60-90 triangle with hypotenuse RS. and legs $\sqrt{6}$ and $3\sqrt{2}$. Area of triangle RTS is $\frac{1}{2}\left(\sqrt{6}\right)\left(6\sqrt{2}\right)$= $6\sqrt{3}$.

The segment of the circle has area

$\frac{1}{3}(24\pi) - 6\sqrt{3}$ = $8\pi - \sqrt{108}$ = $n\pi - \sqrt{m}$.

$m - n$ = 108-8=100