Summary Answers:

1. **258**

2. **– 0.95**

3. **17.78**

4. **80**

5. **1**

6. **0.2302**

7. **29.7072**

8. **403**

9. **1.96**

10. **1050**

11. **150.5952**

12. $\frac{4}{125}$

13. **2.195**

14. $\frac{59}{50}$

Solutions:

1. Answers: A = 5, B = 5, C = 2.58, D = 4, Summary = 258
Solution: Entering the given data values into a list in the calculator and running the *1-Var Stats* program yields a mean of $\bar{x} = 5$, a median of 5, a sample standard deviation of $s_x = 2.58$ (when rounded to hundredths place), and an IQR of 7 – 3 = 4. Thus, the summary answer is 5(5)(2.58)(4) = 258.

2. Answers: Summary = $-$ 0.95
Solution: The correct set of ordered pairs is displayed in the table below:

X	1	2	3	4	5	6	7	8	9	10
Y	9	10	7	8	6	5	3	4	1	2

Entering the above values into a pair of lists in the calculator and running any one of the linear regression programs yields a sample correlation coefficient of r = – 0.95 when rounded to hundredths place. NOTE: Although a boxplot is "an appropriate way to display the distribution of a quantitative data set," it is best paired with "a graphical display of the 5-number summary of a quantitative data set." Hence (8, 4) is best and not (8, 7), as 3 is best paired with 7, i.e. (3, 7).

3. Answers: A = 10.73, B = – 0.95, C = 0, D = 8, Summary = 17.78
Solution: Entering the given values into a pair of lists in the calculator and running any one of the linear regression programs yields a Y-intercept of A = a = 10.73 when rounded to hundredths place and a slope of B = b = – 0.95 when rounded to hundredths place. The two residuals requested are of equal magnitude but opposite in sign, so their sum is 0. Using the *RESID* list function in the calculator to create an entire list of the residuals will reveal this. Otherwise, per the instructions given, the residual for the point (2, 10) is $e = 10 - [-0.95(2) + 10.73] = 1.17$ and the residual for the point (9, 1) is $e = 1 - [-0.95(9) + 10.73] = -1.18$. Their sum is $1.17 + (-1.18) = -0.01$, which rounds to 0 as the nearest integer. D is describing the CENTROID of the scatterplot, which has 8 letters. Therefore, the summary answer is 10.73 + (– 0.95) + 0 + 8 = 17.78.

4. Answers: A = 2, B = 2, C = 10, D = 66, Summary = 80
Solution: A = 2 since there are only two levels of the collar in this experiment, with and without the new chemical. The species of the animal (two levels: dog or cat) and the size (three levels: small, medium, or large) into which they are grouped are both blocking factors in this experiment (and not treatments), therefore B = 2. NOTE: To help distinguish between a treatment and a block, ask: "Can you randomly assign a subject to it?" In general, if the answer is "yes," then it is a treatment. If the answer is "no," then it is a block. For example, one cannot randomly assign an animal to "dog" or "cat" nor to "small, medium, or large," therefore, they are blocks, and not treatments. The following table shows how we can diagram this experiment with the number of animals assigned to each treatment and block combination indicated in each appropriate cell:

Chemical				No Chemical			
Species \ Size	Small	Medium	Large	Species \ Size	Small	Medium	Large
Dog	10	10	10	Dog	10	10	10
Cat	10	10	10	Cat	10	10	10

From the above table, we can see that C = 10. Also, since there is a total of 12 possible treatment and block combinations, there is a total of D = ${}_{12}C_2$ = 66 possible pairwise comparisons. Therefore, the summary answer is: 2 + 2 + 10 + 66 = 80.

5. Answers: A = $\frac{1}{3}$, B = ½, C = ½, D = ¾, Summary = 1
Solution: Using the table above from the previous question, we see there are 10 small cats amongst the 30 cats in the No Chemical group, therefore A = $P(Small\ |(No\ Chemical \cap Cat)) = \frac{10}{30} = \frac{1}{3}$. There are 10 out of 20 total large dogs in the

Chemical group, therefore B = $P(Chemical|(Large \cap Dog)) = \frac{10}{20} = \frac{1}{2}$. There is a total of 80 animals classified as either medium or a cat, of which, 40 are in the No Chemical group. Thus, C = $P(No\ Chemical|(Medium \cup Cat)) = \frac{40}{80} = \frac{1}{2}$. Finally, there is a total of 40 large animals, of which, 30 are either dogs or in the Chemical group. Therefore, D = $P((Chemical \cup Dog)|Large) = \frac{30}{40} = \frac{3}{4}$. The summary answer is: $\frac{\frac{1}{3}\times\frac{3}{4}}{\frac{1}{2}\times\frac{1}{2}} = 1$.

6. Answers: A = $\frac{1}{3}$, B = $\frac{3}{4}$, C = $\frac{3887}{3888}$, D = 0.921, Summary = 0.2302

Solution: Since $\sqrt{2} \approx 1.41421$ and $\pi \approx 3.14159$, $P(\sqrt{2} < X < \pi)$ simplifies to A = $P(2 \le X \le 3) = \frac{2}{6} = \frac{1}{3}$. The probability distribution for the sum on two dice is given below (with the common denominator of 36, for convenience):

Y = Sum	2	3	4	5	6	7	8	9	10	11	12
P(Y)	1 / 36	2 / 36	3 / 36	4 / 36	5 / 36	6 / 36	5 / 36	4 / 36	3 / 36	2 / 36	1 / 36

Therefore, B = $P(4 \le Y < 10) = P(4 \le Y \le 9) = \frac{3+4+5+6+5+4}{36} = \frac{27}{36} = \frac{3}{4}$. Part C is not as difficult as it may seem at first even though it is for the sum on five dice. The complement rule simplifies things greatly: C = $P(6 \le F \le 29) = 1 - P(F = 1 \cup F = 30) = 1 - \frac{2}{6^5} = \frac{3887}{3888}$. The hint for part D indicates that one should use a normal distribution approximation to this situation in either one of two ways. First, the mean (or expected value) for the sum of the 100 dice is $\mu_H = E(H) = 3.5 + 3.5 + 3.5 \ldots 3.5 = 100(3.5) = 350$ and the variance is $\sigma_H^2 = \frac{35}{12} + \frac{35}{12} + \frac{35}{12} \ldots \frac{35}{12} = 100\left(\frac{35}{12}\right) = \frac{875}{3}$. Using the normal approximation, we get $D = P(320 \le H \le 380) \approx normalcdf\left(320, 380, 350, \sqrt{\frac{875}{3}}\right) \approx 0.921$ when rounded to thousandths place. Also, we can use a sampling distribution for the sample mean with $\mu_{\bar{H}} = E(\bar{H}) = 3.5$ and $\sigma_{\bar{X}} = \sqrt{\frac{\frac{35}{12}}{100}} = \sqrt{\frac{7}{240}}$. This way, $D = P(3.20 \le \bar{H} \le 3.80) \approx normalcdf\left(3.20, 3.80, 3.5, \sqrt{\frac{7}{240}}\right) \approx 0.921$ as well. Isn't the CLT beautiful? The summary answer is $\frac{1}{3} \times \frac{3}{4} \times \frac{3887}{3888} \times (0.921) \approx 0.2302$ when rounded to four decimal places.

7. Answers: A = 0, B = 28.2843, C = 0.5, D = 0.9229, Summary = 29.7072

Solution: $A = \mu_{4x-2y} = E(4X - 2Y) = 4(10) - 2(20) = 0$. $B = \sigma_{4x-2y} = SD(4X - 2Y) = \sqrt{16(25) + 4(100)} = \sqrt{800} \approx 28.2843$ rounded to four decimal places. $C = P[(4X - 2Y) < 0] = 0.5$ since it is asking for the probability that a normally distributed random variable takes on a value less than the mean. $D = P[-50 < (4X - 2Y) < 50] = normalcdf(-50, 50, 0, 28.2843) \approx 0.9229$ when rounded to four decimal places. Summary = 0 + 28.2843 + 0.5 + 0.9229 = 29.7072.

8. Answer: A = 129, B = 0.033836, C = 2.088, D = 1, Summary = 403

Solution: The expected value of the number of AP Statistics Exam takers who pass the exam under the assumption that the pass rate in Florida is the same as the global pass rate is A = 200(0.645) = 129. The standard deviation of the sampling distribution of the sample proportion for this test is given by B = $\sigma_{\hat{p}} = \sqrt{\frac{p_0 q_0}{n}} = \sqrt{\frac{(0.645)(1-0.645)}{200}} \approx 0.033836$ when rounded to six decimal places. Entering P_0 = 0.645, x = 200(0.715) = 143, n = 200, into the *1-PropZTest* program in the calculator yields a test statistic of $Z \approx 2.069$ when rounded to thousandths place and a p-value of $p \approx 0.019$ when rounded to thousandths place. Those who use the formula will obtain the same results so long as they use the rounded result from part A: $Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.715 - 0.645}{0.033836} \approx 2.069$, when rounded to thousandths place, and the p-value is given by $p = normalcdf(2.069, 9999999, 0, 1) \approx 0.019$, when rounded to thousandths place as well. Also, using the chart with Z rounded to 2.07 will yield the same result for the p-value. Hence, C = 2.069 + 0.019 = 2.088. Finally, since the p-value of the test from part C is less than 0.05, we can reject the null of H_0: P = 0.645 and support H_A: P > 0.645 at the 5% level of significance. This makes D = 1. The summary answer is 129(0.033836 + 2.088 + 1) = 402.716844 which rounds up to 403 as the nearest integer. NOTE: The reason the final summary answer is rounded to the nearest integer is in the interest of fairness to those students who may have slightly differing results in the parts due to rounding, yet will still earn credit for conceptually knowing how to solve the problem.

9. Answers: A = 0.032, B = 0.633, C = 0.797, D = 0**,** Summary = 1.96
Solution: The standard error of the sampling distribution of the sample proportion for this confidence interval is given by A = $SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{(0.715)(1-0.715)}{200}} \approx 0.032$ when rounded to thousandths place. The critical Z* for this 99% confidence interval is Z* = $invNorm(0.995, 0.1) \approx 2.576$ when rounded to thousandths place. Thus, the lower limit of the 99% confidence interval is B = 0.715 – 2.576(0.032) = 0.632568 which rounds to 0.633 in thousandths place. Also, the upper limit of the 99% confidence interval is C = 0.715 + 2.576(0.032) = 0.7974332 which rounds to 0.797 in thousandths place. Using the *1-PropZInt* program with x = 200(0.715) = 143, n = 200, and C-Level: 99% yields the interval (0.63278, 0.79722) which rounds to the same correct results for parts B and C. Since the 99% confidence interval captures the null hypothesis value of H_0: P = 0.645, we lack statistically significant evidence to reject it and fail to support H_A: P ≠ 0.645 at the 1% level of significance. Hence, D = 0. Summary = 10(0.032 – 0.633 + 0.797 + 0) = 1.96.

10. Answers: A = 0.0884, B = 4.186, C = 199, D = 1, Summary = 1050
Solution: This scenario is a one-sample t-test of H_0: μ = 2.86 versus H_A: μ > 2.86. The standard error of the sampling distribution of the sample mean of this test is given by $A = SE(\bar{X}) = \frac{s_x}{\sqrt{n}} = \frac{1.25}{\sqrt{200}} \approx 0.0884$ when rounded to the nearest ten-thousandth. The tests statistic is B = $t = \frac{\bar{X}-\mu_0}{\frac{s_x}{\sqrt{n}}} = \frac{3.23-2.86}{0.0884} \approx 4.186$ when rounded to the nearest thousandth. The p-value is $tcdf(4.186, 999999, 199) \approx 0.0000213$. Thus, C = 199 and D = 1 for "yes" since the p-value is less than 1%. Entering the given values into the *T-Test* program in the calculator produces the same results, when rounded accordingly. Summary = 199(0.0884 + 4.186 + 1) = 1049.6056 which rounds to 1050 as the nearest integer.

11. Answers: A = 0.1743, B = 95, C = 216, D = 553, Summary = 150.5952
Solution: The margin of error for the given confidence interval is A = $\frac{3.4043-3.0557}{2} = 0.1743$. The critical t-score for the interval is found by solving the margin of error formula for t^*: $t^* = \frac{ME}{s_x/\sqrt{n}} = \frac{0.1743}{1.25/\sqrt{200}} \approx 1.972$ when rounded to thousandths place. Using $tcdf(-1.972, 1.972, 199)$ will result in the correct level of confidence of B = 95% when rounded to the nearest integer percent. NOTE: Rounding the critical t^* to at least hundredths place prior to using the *tcdf* will yield the correct confidence level when rounded to the nearest integer. Alternatively, trial-and-error with various levels of confidence in the *TInterval* program will lead to the given confidence interval limits of 3.0557 and 3.4043 when using 95% confidence. The critical Z^* for a 94% confidence interval is Z^* = $invNorm(0.97, 0, 1) \approx 1.88$ when rounded to hundredths place. So, the minimum sample size required to estimate the mean in question is given by $n \geq \left(\frac{Z^*\sigma}{ME}\right)^2 = \left(\frac{1.88\times1.25}{0.16}\right)^2 \approx 215.72 \rightarrow 216$ = C. Likewise, the minimum sample size required to estimate the proportion in question is given by $n \geq \hat{p}\hat{q}\left(\frac{Z^*}{ME}\right)^2 = (0.5)(0.5)\left(\frac{1.88}{0.04}\right)^2 \approx 552.24 \rightarrow 553$ = D. Summary = 0.1743(95 + 216 + 553) = 150.5952

12. Answers: A = $\frac{1}{10}$, B = $\frac{3}{5}$, C = $\frac{8}{15}$, D = 1, Summary = $\frac{4}{125}$
Solution: A = $P(A^C \cap B^C) = \frac{40}{400} = \frac{1}{10}$, B = $P(A \cup B^C) = \frac{200+100-60}{400} = \frac{240}{400} = \frac{3}{5}$, and C = $P(A^C \mid B) = \frac{160}{300} = \frac{8}{15}$. D is the exhaustive set of all possible outcomes, thus D = $P[(A \cap B) \cup (A^C \cap B) \cup (A \cap B^C) \cup (A^C \cap B^C)] = \frac{140+160+60+40}{400} = \frac{400}{400} = 1$. Summary = $\frac{1}{10} \times \frac{3}{5} \times \frac{8}{15} \times 1 = \frac{4}{125}$.

13. Answers: A = 0.35, B = 0.75, C = 5.5, D = 2.18, Summary = 2.195
Solution: A = $P(4 \leq x < 7) = P(4) + P(5) + P(6) = 0.05 + 0.20 + 0.10 = 0.35$.
B = $P(x > 3 \mid x \; is \; odd) = P(x = 5 \; or \; 7) \; / \; P(x = 3, 5, or \; 7) = (0.20 + 0.10) \; / \; (0.10 + 0.20 + 0.10) = 0.75$.
For parts C and D, enter the outcomes for X and corresponding probabilities into a pair of lists in the calculator and run the *1-VarStats* program on the pair of lists. The resulting mean and standard deviation are 5.5 and 2.18, respectively.
Summary: the mean of set {0.35, 0.75, 5.5, 2.18} is (0.35 + 0.75 + 5.5 + 2.18) / 4 = 2.195.

14. Answers: A = $\frac{1}{10}$, B = $\frac{9}{10}$, C = $\frac{9}{100}$, D = $\frac{9}{100}$, Summary = $\frac{59}{50}$
Solution: Set S has a total of 10(10) = 100 elements, 90 of which are real numbers and 10 of which are not (when b = 0). So, in order for an element of set S to be undefined, we need b = 0. This means a can be any of the ten digits which makes this probability A = $\frac{1}{10}$. All elements of set S are rational, except when b = 0, as in part A. Thus, B = $1 - \frac{1}{10} = \frac{9}{10}$.
In order for an element of set S to be equal to 0, we need a = 0 and b ≠ 0. So, b can be any of the nine non-zero digits while a is fixed at 0. Thus, C = $\frac{1}{10} \times \frac{9}{10} = \frac{9}{100}$. In order for an element of set S to be equal to 1, we need a = b and neither equal to 0. This can happen in nine ways. So, D = $\frac{9}{100}$. Summary = $\frac{1}{10} + \frac{9}{10} + \frac{9}{100} + \frac{9}{100} = \frac{59}{50}$.