SOLUTIONS

1. E

2. B

3. C

4. B

5. C

6. A

7. B

8. D

9. A

10. C

11. B

12. D

13. B

14. A

15. C

16. A

17. D

18. B

19. D

20. A

21. C

22. D

23. C

24. B

25. C

26. B

27. D

28. C

29. B

30. A

1. Pi is an irrational number. Choice A is an approximation. Choices B, C, and D are wrong because irrationals numbers cannot be expressed as a ratio.

Hence the answer is E

2. $x - \sqrt{4-x} = 4$

$x - 4 = \sqrt{4-x}$

$x^2 - 8x + 16 = 4 - x$

$x^2 - 7x + 12 = 0$

$(x-3)(x-4) = 0$

$x \neq 3 \rightarrow x = 4$

Hence the answer is B

3. $A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(6-2)[(13.5-1.5)+(9.5-5.5)]$

$A = 32$

Hence the answer is C

4. $(4)(-2) - (5)(-6) = 22$

Hence the answer is B

5. $a - b = 6$ and $\frac{a}{b} = 2$

$a - \frac{a}{2} = 6$

$a = 12 \rightarrow b = 6$

$\sqrt{144-36} = 6\sqrt{3}$

Hence the answer is C

6. I. $f(x) \neq f(-x)$

II. $f(x) = f(-x)$

III. $f(x) \neq f(-x)$

Hence the answer is A

7. $|4+8i| = \sqrt{16+64} = 4\sqrt{5}$

Hence the answer is B

8. I. True

II. True

III. True

IV. False

Hence the answer is D

9. $\frac{1-7x}{3x^2+7x-6} = \frac{A}{x+3} + \frac{B}{3x-2}$

$\frac{1-7x}{3x^2+7x-6} = \frac{A(3x-2)+B(x+3)}{3x^2+7x-6}$

$(3A+B)x + (3B-2A) = 1-7x$

$3A + B = -7$ and $3B - 2A = 1$

$2A = 3(-7-3A) - 1$

$A = -2$

$B = -7 - 3(A) = -1$

$A + B = -3$

Hence the answer is A

10. Use the substitution $a = 2\sqrt{8+a}$

$a^2 = 4(8+a)$

$a^2 - 4a - 32 = 0$

$(a-8)(a+4) = 0$

a cannot be negative, $a = 8$

Hence the answer is C

11. $5(x^2-4x)-4(y^2+10y)=100$

$5(x-2)^2-4(y+5)^2=100+20-100$

$\frac{(x-2)^2}{4}-\frac{(y+5)^2}{5}=1$

The foci are located at $\left(2\pm\sqrt{5+4},-5\right)$ or $(-1,-5)$ and $(5,-5)$

Hence the answer is B

12. $\left(\sqrt{12+\sqrt{128}}\right)^2=\left(\sqrt{x}+\sqrt{y}\right)^2$

$12+\sqrt{128}=x+y+2\sqrt{xy}$

$x+y=12$ and $2\sqrt{xy}=\sqrt{128}$ or $xy=32$

$x(12-x)=32$

$x^2-12x+32=0$

$(x-4)(x-8)=0$

If $x+y=12$, then $x=4$ because $y>5$

$|y-x|-x=0$

Hence the answer is D

13. $f(x)=x+2$ → $x=f^{-1}(x)+2$

$f^{-1}(x)=x-2$

$g(x)=x^2+9$ → $x=\left(g^{-1}(x)\right)^2+9$

$g^{-1}(x)=\sqrt{x-9}$

$f(23)=25$

$g^{-1}\big(f(23)\big)=\sqrt{25-9}=4$

$f^{-1}\left(g^{-1}\big(f(23)\big)\right)=4-2=2$

Hence the answer is B

14. $4^3<\log_4 123<4^4$

The characteristic is 3

Hence the answer is A

15. $b^2 - 4ac = 144 - 4(1)(-24)$

$b^2 - 4ac = 240$

Hence the answer is C

16. Using synthetic division:

$$\begin{array}{r|rrrrr} & 1 & -3 & 0 & 5 & 8 \\ -2 & & -2 & 10 & -20 & 30 \\ \hline & 1 & -5 & 10 & -15 & 38 \end{array}$$

The remainder is 38

Hence the answer is A

17. $6^{x^2+5} = 216^{x+5}$

$6^{x^2+5} = 6^{3(x+5)}$

$x^2 + 5 = 3x + 15$

$x^2 - 3x - 10 = 0$

$x = \{-2,5\}$

Hence the answer is D

18. This is the perpendicular line, to the line segment formed by $(-7,5)$ and $(9,-3)$, that passes through the midpoint of the line segment formed by $(-4,5)$ and $(9,-3)$

$$m_{segment} = \frac{-3-5}{9-(-7)} = -\frac{1}{2}$$

$$midpoint\text{: } \left(\frac{-7+9}{2}, \frac{-3+5}{2}\right) \text{ or } (1,1)$$

Using point-slope form: $y - 1 = 2(x - 1)$

$$y = 2x - 1$$

Hence the answer is B

19. $6y + 3z = 48 \rightarrow 2y + z = 16$ Rewrite Eq 2

Subtract Eq 2 from Eq 3 $\rightarrow x - y = 1$ Eq A

Subtract Eq 2 from Eq 1 $\rightarrow 3x - 7y = -13$ Eq B

Multiply Eq A by -3 and add to Eq B $\rightarrow -4y = -16$

$y = 4$

$x - y = 1 \rightarrow x = 5$

$x + y + z = 17 \rightarrow z = 8$

$x - y^2 + z^2 = 53$

Hence the answer is D

20. $x^2 + y^2 + 4x - 8y = 5$

$(x + 2)^2 + (y - 4)^2 = 25$

$d = \sqrt{(-2 - 7)^2 + \left(4 - (-8)\right)^2} = 15$

Hence the answer is A

21. $\frac{1}{(\sqrt{3}+\sqrt{12})+\sqrt{18}} = \frac{1}{3\sqrt{3}+3\sqrt{2}}$

$\frac{1}{3\sqrt{3}+3\sqrt{2}} \times \frac{3\sqrt{3}-3\sqrt{2}}{3\sqrt{3}-3\sqrt{2}} = \frac{3\sqrt{3}-3\sqrt{2}}{27-18}$

$\frac{1}{3\sqrt{3}+3\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3}$

Hence the answer is C

22. $x = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2$

$f(2) = (2)^2 - 4(2) - 12 = -16$

Hence the answer is D

23. $0.456 + 0.788 - 0.352 = \frac{456}{1000} + \frac{788}{1000} - \frac{352}{1000} = \frac{892}{1000}$

$\frac{1}{0.456+0.788-0.352} = \frac{1000}{892} = \frac{250}{223}$

Hence the answer is C

24. $y = \frac{1}{4p}(x-h)^2 + k$

$y = \frac{1}{4(6-2)}(x+3)^2 + 2$

$y = \frac{1}{16}(x+3)^2 + 2$

Hence the answer is B

25. $f(x) = x^2 + 3x + 3$

$(x^2 + 3x) = -3$

$x^2 + 3x + \frac{9}{4} = \frac{9}{4} - 3$

$\left(x + \frac{3}{2}\right)^2 = -\frac{3}{4} \rightarrow x = -\frac{3}{2} \pm i\frac{\sqrt{3}}{2}$

$a^2 + b^2 + c^2 + d^2 = \left(-\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{3}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = 6$

Hence the answer is C

26. The remainder when any number is divided by 10 is the units digit

The units digits follows the pattern of powers of 3: 3,9,27,81,243, ...

1st power: 3

2nd power: 9

3rd power: 7

4th power: 1

5th power: 3

Repeats after multiples of 4

$23^{43} = 23^{4(10)+3} \rightarrow$ The units digit is 7

Hence the answer is B

27. $c^2 = a^2 - b^2$ and $e = {}^{c}/_{a}$

The foci are along the *y* direction, major axis follows the *y* direction; cannot be A or C

For answer choice B: Center is $(-5,3)$ and $c = \sqrt{\frac{9}{4} - \frac{7}{4}} = \frac{\sqrt{2}}{2}$

For answer choice D: Center is $(-5,3)$ and $c = \sqrt{16 - 7} = 3$

Hence the answer is D

28. $(3 - i)(2 + 9i) = 6 + 27i - 2i + 9 = 15 + 25i$

Hence the answer is C

29. $H_{mean} = \frac{2}{\frac{1}{6}+\frac{1}{8}} = \frac{2}{\frac{14}{48}} = \frac{2(48)}{14} = \frac{48}{7}$

Hence the answer is B

30. $4x - 5y = 13$

$y = \frac{4}{5}x - \frac{13}{5}$

The slope of the perpendicular line is the negative reciprocal or $-{}^{5}/_{4}$

Hence the answer is A