Question Number	Answer	Question Number	Answer
1	$\frac{1}{3}$	**9**	**2.53**
2	**30240**	**10**	**0.326**
3	**16.92**	**11**	-2.25
4	**781403**	**12**	$\frac{828}{4165}$
5	**1.738**	**13**	**34**
6	$\frac{121}{12}$	**14**	**100.7744**
7	**16.83**	**15**	**123.3**
8	**-0.116**		

1. 1/3

A) $E(3X+5Y)=3E(X)+5E(Y)=3*10+5*5=55$

B) $E(3Y+X)=3E(Y)+E(X)=3*5+10=25$

C) $E(2X+2Y)=2E(X)+2E(Y)=2*10+2*5=30$

D) $Var(3X-2Y)=3^2Var(X)+2^2Var(Y)-2*3*2*\rho*SD(X)*SD(Y)=289.6$

$289.6=9*36+4*4-2*3*2*6*2*\rho$

$-50.4=-2*3*2*6*2*\rho$

$\rho=0.35$

E) $Slope=\beta_1=0.35*\frac{SD(Y)}{SD(X)}=0.35*\frac{2}{6}=\frac{7}{60}$

Summary: $A+\frac{E}{D}-(C+B)=55+\frac{\frac{7}{60}}{0.35}-(30+25)=\frac{1}{3}$

2. 30240

<u>Crossword Solutions</u>

2-Across: STRATIFIED, 5-Across: CONVENIENCE, 1-Down: SIMPLERANDOM, 3-Down: CLUSTER, 5-Down: SYSTEMATIC

Number of Distinct Permutations in "RILENNEET" $=\frac{9!}{3!*2!}=30240$

3. 16.92

Where W denotes a win and L denotes a loss, to make a game fair, $5 $*P(W)-MoneyLost*P(L)=0$, thus making our expected profit 0. If $5*P(W)=MoneyLost*P(L)$, then $MoneyLost=\frac{5*P(W)}{P(L)}=\frac{5*P(W)}{1-P(W)}$. We shall use this expression for the parts below.

A) There are 20 multiples of three on [1,60]. $P(W)=\frac{1}{3}$ and $MoneyLost=\frac{5*\frac{1}{3}}{\frac{2}{3}}=\$2.50$

B) There are 8 multiples of seven on [1,60]. $P(W)=\frac{2}{15}$ and $MoneyLost=\frac{5*\frac{2}{15}}{\frac{13}{15}}=\$0.77$

C) There are 17 primes on [1,60]. $P(W)=\frac{17}{60}$ and $MoneyLost=\frac{5*\frac{17}{60}}{\frac{43}{60}}=\$1.98$

D) There are 42 composite integers on [1,60]. $P(W)=\frac{7}{10}$ and $MoneyLost=\frac{5*\frac{7}{10}}{\frac{3}{10}}=\$11.67$

NOTE: The integer 1 is neither prime nor composite!

Summary: $A+B+C+D=16.92$

4. 781403

This question is intended to be done arithmetically as the numbers are too large for most calculators to easily make into a reduced fraction.

A) $binompdf(4{,}0.75{,}3) = \frac{27}{64}$

B) $geometpdf(0.75{,}2) + geometpdf(0.75{,}3) = \frac{15}{64}$

C) $0.75 * binompdf(4{,}0.75{,}2) = \frac{81}{512}$

$$P(at\ least\ 1) = 1 - P(none) = 1 - \left(1 - \frac{27}{64}\right)\left(1 - \frac{15}{64}\right)\left(1 - \frac{81}{512}\right) = 1 - \frac{37}{64} * \frac{49}{64} * \frac{431}{512} = 1 - \frac{37 * 49 * 431}{64 * 64 * 512}$$

$$= \frac{64 * 64 * 512 - 37 * 49 * 431}{64 * 64 * 512} = \frac{1315749}{2097152}$$

Since we know that we only have powers of two in the denominator and our numerator is odd, we can conclude that that the fraction is fully reduced.

Summary: $2097152 - 1315749 = 781403$

5. 1.738

Below is the table with totals filled in:

	Republican (R)	Democrat (D)	Third Party (T)	Total
Freshman (F)	370	297	58	725
Sophomore (So)	316	309	77	702
Junior (J)	271	334	90	695
Senior (Se)	232	368	81	681
Total	1189	1308	306	2803

A) $P(R \cup Se) = \frac{370+316+271+232+368+81}{2803} = 0.584$

B) $P(D \cap F) = \frac{297}{2803} = 0.106$

C) $P(So' \mid T) = \frac{58+90+81}{306} = 0.748$

D) $P(\,((F \cup J) \cap D) \mid So') = \frac{297+334}{725+695+681} = 0.300$

Summary: $A + B + C + D = 1.738$

6. $\frac{121}{12}$

Because we are dealing with a continuous uniform distribution, its expected value is $\frac{A+B}{2}$.

$$Var(X) = E(X^2) - E(X)^2 = \frac{A^2 + AB + B^2}{3} - \left(\frac{A+B}{2}\right)^2 = \frac{A^2 + AB + B^2}{3} - \frac{A^2 + 2AB + B^2}{4}$$

$$= \frac{4(A^2 + AB + B^2) - 3(A^2 + 2AB + B^2)}{12} = \frac{4A^2 + 4AB + 4B^2 - 3A^2 - 6AB - 3B^2}{12} = \frac{A^2 - 2AB + B^2}{12}$$

$$Var(X) = \frac{(B - A)^2}{12}$$

For the given distribution bounded by $[226, 237]$, $Var = \frac{(237-226)^2}{12} = \frac{121}{12}$

7. 16.83

A) There are 2 experimental factors in this experiment: fertilizer brand and nitrate content. So, A = 2.

B) Since we are attempting to control for the confounding factors of soil type and other environmental factors by conducting the experiment in 6 different agricultural states, each of the 6 states is a block. So, B = 6.

C) Fertilizer brand has 3 levels: X, Y, and Z; and nitrate content had 7 levels: 4 ppm, 8 ppm, 12 ppm, 16 ppm, 20 ppm, 24 ppm, and 28 ppm; there are $3 \times 7 = 21$ total treatments. As noted above, the different states are blocks and therefore, not treatments since the experiment is essentially replicated within each state (or block). So, C = 21.

D) Some amount of confounding is always present in every experimental design, which is why we should try to control confounding factors via such methods as blocking or other forms of control. Hence, these concepts are certainly present in this experiment. Sufficient replication in the form of a sufficient number of experimental units randomly assigned to each treatment is also a requirement of any good experiment. However, since we are experimenting on plant growth based on the effect of fertilizer type, blinding is neither possible nor necessary. So, D = 3.

Summary: Entering the numbers 2, 6, 21, and 3 into a list in the calculator and running 1-Var Stats we obtain a sample mean of $\bar{x} = 8$ and a sample standard deviation of $s_x = 8.83$ and so their sum is 16.83.

8. -0.116

A) $P(k = 3) = e^{-4}\frac{4^3}{3!} = 0.195$

B) $P(k = 0) = e^{-0.4}\frac{0.4^0}{0!} = 0.670$

C) -1, because k must take on integer values, so the Poisson Distribution is not appropriate.

D) $P(k > 3) = 1 - (e^{-1}\frac{1^0}{0!} + e^{-1}\frac{1^1}{1!} + e^{-1}\frac{1^2}{2!} + e^{-1}\frac{1^3}{3!}) = 0.019$

Summary: $A + B + C + D = -0.116$

9. 2.53

The question directions state Normal Approximations to the Binomial Distribution must be used, so calculations involving binomcdf or binompdf should not be accepted.

A) $normalcdf(79700, \infty, 160000 * 0.5, \sqrt{160000 * 0.5 * 0.5}) = 0.93$

B) $normalcdf(79700, 80300, 160000 * 0.5, \sqrt{160000 * 0.5 * 0.5}) = 0.87$

C) $P(X > 80300 \mid X > 79700) = \frac{normalcdf(80300, \infty, 160000*0.5, \sqrt{160000*0.5*0.5})}{normalcdf(79700, \infty, 160000*0.5, \sqrt{160000*0.5*0.5})} = 0.07$

D) Because we are told the first 70000 trials have been observed, we are now assessing the remaining 90000 instead of the original 160000 in our Normal Probability calculation. $80400 - 35500 = 44900$ and $80700 - 35500 = 45200$.

$P(80400 < X < 80700) = normalcdf(44900, 45200, 90000 * 0.5, \sqrt{90000 * 0.5 * 0.5}) = 0.66$

Summary: $A + B + C + D = 2.53$

10. 0.326

A) $\frac{\binom{9}{4}\binom{9}{4}\binom{9}{4}}{\binom{27}{12}} = 0.115$

B) $\frac{\binom{9}{8}\binom{9}{4}\binom{9}{8}}{\binom{27}{20}} = 0.011$

C) $\frac{\binom{9}{4}\binom{9}{4}\binom{9}{8}}{\binom{27}{16}} + \frac{\binom{9}{8}\binom{9}{4}\binom{9}{4}}{\binom{27}{16}} = 0.022$

D) $\frac{\binom{9}{4}\binom{9}{4}\binom{9}{8}}{\binom{27}{16}} + \frac{\binom{9}{5}\binom{9}{4}\binom{9}{7}}{\binom{27}{16}} + \frac{\binom{9}{6}\binom{9}{4}\binom{9}{6}}{\binom{27}{16}} + \frac{\binom{9}{7}\binom{9}{4}\binom{9}{5}}{\binom{27}{16}} + \frac{\binom{9}{8}\binom{9}{4}\binom{9}{4}}{\binom{27}{16}} = 0.178$

Summary: $A + B + C + D = 0.326$

11. -2.25

Using 2-Var Stats, $E(X) = 1.5$ and $E(Y) = -1.5$

A	B	C	$X = -A + 2B + 2C$	$Y = A - 2B - 2C$	$(x - E[X])(y - E[y])$
0	0	0	0	0	-2.25
0	0	1	2	-2	-0.25
0	1	0	2	-2	-0.25
0	1	1	4	-4	-6.25
1	0	0	-1	1	-6.25
1	0	1	1	-1	-0.25
1	1	0	1	-1	-0.25
1	1	1	3	-3	-2.25

Using 1-Var Stats, $E[(x - E[X])(y - E[y])] = -2.25$

12. $\mathbf{\frac{828}{4165}}$

A) $P(5\ of\ a\ kind) = \frac{1}{\binom{53}{5}}$. There is only one possible 5 of a kind in our modified deck of cards.

B) $P(Royal\ Flush) = \frac{5}{\binom{53}{5}}$. There are normally 4 distinct royal flushes in a deck of 52 cards, but the second Ace of Spades grants us a second royal flush of spades.

C) $P(Any\ given\ hand\ with\ both\ Aces\ of\ Spades) = \frac{\binom{2}{2}\binom{51}{3}}{\binom{53}{5}}$. We wish to choose our two desired Aces and then 2 other arbitrary cards of the remaining 51.

D) $P(4\ of\ a\ kind) = \frac{\binom{choosing\ a\ 4\ of\ a\ kind}{of\ all\ cards\ except\ aces} + \binom{choosing\ a\ 4\ of}{a\ kind\ of\ aces}}{\binom{53}{5}} = \frac{\binom{12}{1}\binom{4}{4}\binom{49}{1} + \binom{1}{1}\binom{5}{4}\binom{48}{1}}{\binom{53}{5}}$

$$\text{Summary: } \frac{BD}{AC} = \frac{\frac{5}{\binom{53}{5}} * \frac{\binom{12}{1}\binom{4}{4}\binom{49}{1} + \binom{1}{1}\binom{5}{4}\binom{48}{1}}{\binom{53}{5}}}{\frac{1}{\binom{53}{5}} * \frac{\binom{2}{2}\binom{51}{3}}{\binom{53}{5}}} = \frac{5*828}{20825} = \frac{828}{4165}$$

13. 34

The table shows the number of viewers for each movie using the given ratio 3:2:1

"The Matrix"	"The Matrix Reloaded"	"The Matrix Revolutions"	Total
51	34	17	102

A) Because we are sampling the entire population, we are taking a census. There are 5 distinct letters in the word census.

B) 34, see table above.

C) $\frac{51}{102} = \frac{1}{2}$

D) $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD - CB = 153, 5 * D - \frac{1}{2} * 34 = 153, D = 34$

14. 100.7744

$Residual = y - \hat{y}$

A) $Residual = 4600 - (-3520 * 1.5 + 10040) = -160$

B) $Residual = 1500 - (-3520 * 2.5 + 10040) = 260$

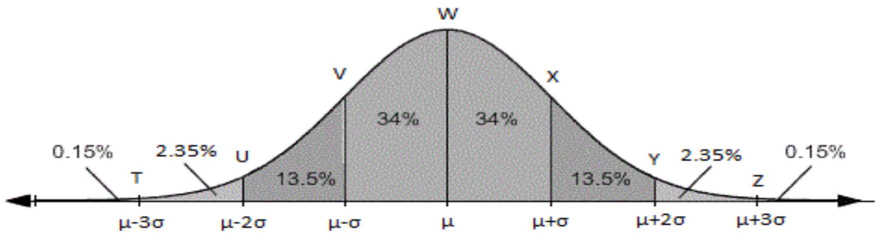
C) $Slope = r\frac{S_y}{S_x}, -3520 = r * \frac{2000}{0.5}, r = -0.88$

The proportion of explained variance $= r^2 = (-0.88)^2 = 0.7744$

Summary: $A + B + C = 100.7744$

15. 123.3

This is a primarily logic-based question. Chop the normal distribution into sections with labelled empirical percentages to make this question easier. The points an integer number of standard deviations from the mean of any given Normal Distribution have also been marked in the picture below for ease of explanation.



- Lollipop Distribution
  - With the first clue, we are told that 45.8 is an upper bound while the interval $[58.6,65]$ also comprises some area of the distribution. 16% of the distribution can be seen as the interval (U, V) or $(\text{X}, \text{Y}) + (\text{T}, \text{U})$ or $(\text{Y}, \text{Z}) + (-\infty, \text{T})$ or (Z, ∞). 45.8 must either be point U or T because 45.8 is an upper bound and $P(L < 45.8) \neq 0.16$ so 45.8 cannot be V.
    - If 45.8 is U, then 58.6 must be X and 65 must be Y to capture 16% of the distribution.
      - Evaluating $Y - X$, we get $\sigma = 6.4$.
      - However, while the distance from U to X would be $3\sigma = 19.2$ according to our standard deviation, $58.6 - 45.8 \neq 12.6$ so this does not work.
    - If 45.8 is T, then 58.6 must be X and 65 must be Z to capture 16% of the distribution.
      - Evaluating $\frac{Z-X}{2}$, we get $\sigma = 3.2$.
      - Since the distance from T to X should be $4\sigma = 12.8$, and $58.6 - 45.8 = 12.8$, this works.
    - $\sigma = B = 3.2$ and $\mu = A = 55.4$
  - The second clue is not necessary to solve the problem, but we can see that 52.2 being point V reinforces our solution.
- Cupcake Distribution
  - The third clue establishes 51.9 as an upper bound for 16% of the distribution, meaning 51.9 must be V.
  - The fourth clue establishes 77.5 as a lower bound for some small percentage of the distribution as well as 64.7 as a lower bound for another percentage of the distribution. We may notice that 64.7 is the midpoint between 51.9 and 77.5. Since that is the case, 64.7 could be W and 77.5 would be X, or 64.7 could be X and 77.5 would be Z.
    - If 64.7 is W, then $\frac{P(C>77.5)}{P(C>64.7)} = \frac{.16}{.5} = \frac{8}{25} \neq \frac{3}{320}$, so this does not work.
    - If 64.7 is X, then $\frac{P(C>77.5)}{P(C>64.7)} = \frac{.0015}{.16} = \frac{3}{320}$, so this works.
    - $\sigma = D = 6.4$ and $\mu = C = 58.3$

Summary: $A + B + C + D = 55.4 + 3.2 + 58.3 + 6.4 = 123.3$