January Regional Statistics Team: Question #1

Consider the random variables X and Y. X has a mean of 10 and a standard deviation of 6. Y has a mean of 5 and a standard deviation of 2. Evaluate each of the following expressions.

A) The mean of $3X + 5Y$.

B) The mean of $3Y + X$.

C) The mean of $2X + 2Y$.

D) Find the correlation coefficient between X and Y if the variance of $3X - 2Y$ is 289.6.

E) Using the correlation coefficient from part D above, find the slope of the linear relationship between Y and X where Y is the response variable and X is the explanatory variable.

Evaluate the expression $A + \frac{E}{D} - (C + B)$ as a simplified fraction.

January Regional Statistics Team: Question #2

Complete the empty crossword puzzle of the methods for sampling students from a large high school then find the number of distinct permutations of the letters in the boldly bordered boxes in the puzzle. Count each boxed letter where there is an intersection at a box only once and not twice. Assume each clue is in the form "______ sample."

Across:

2. Segregate students by grade level and then obtain a SRS of students from each grade level.

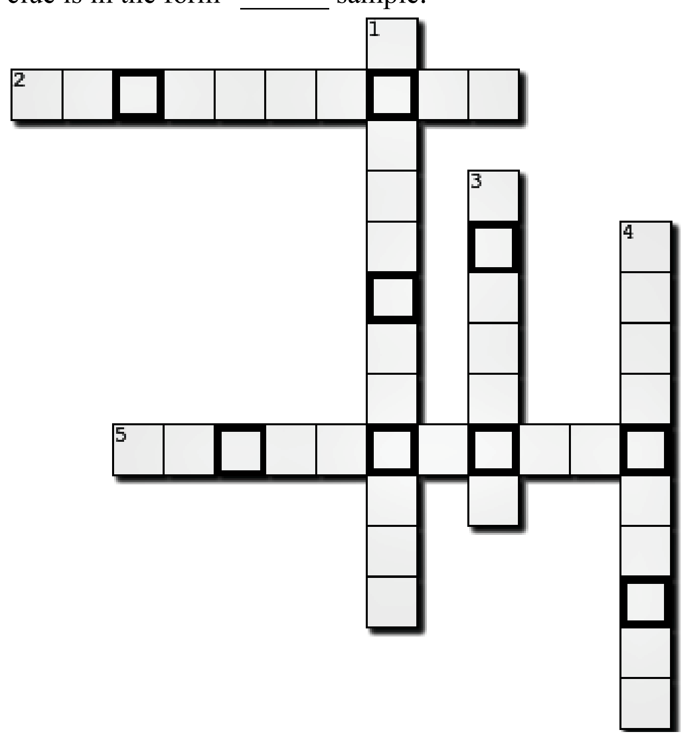
5. Sample the first 50 students I see when I walk in the front door.

Down:

1. Randomly select a subset of 50 students from the school such that all possible subsets of 50 students are equally likely to be the actual subset of 50 students chosen (two words but do not treat the space as a letter).

3. Randomly select a few classrooms at the school such that each classroom is presumably representative of the school as a whole and then sample all students within the selected classrooms.

4. Start with a randomly selected student from within the first 10 students in an alphabetized list of all students at the school. Then, select every tenth student thereafter to obtain the rest of the sample.



January Regional — Statistics Team: Question #3

The Wheel of Fun has been introduced at your local casino. The wheel is filled with positive integers numbered 1-60, inclusive, each with equal probability of occurring. To play, you bet on specific sets of integers on the wheel. If the wheel lands on any element of your set, you win 5 dollars. Otherwise, you lose a certain amount of money such that every game on the Wheel of Fun shall be fair. Find the amount of money lost if you don't land on each of the winning sets below. Round the final answer for each part and the summary answer below to the nearest cent.

A) Multiples of 3

B) Multiples of 7

C) Prime Integers

D) Composite Integers

What is $A + B + C + D$?

January Regional — Statistics Team: Question #4

I own three special coins: A, B, and C. Unlike fair coins, the coins are weighted such that each coin will turn up heads 75% of the time and tails 25% of the time when flipped. Calculate the probability of obtaining each of the following events on each respective coin.

Coin A: I get exactly 3 heads in 4 flips.

Coin B: It takes me either 2 or 3 flips to see my first head.

Coin C: I get my third head on my 5^{th} flip.

The probability that at least one of the above events occurs on the three independent coins can be expressed as a simplified fraction in the form $\frac{A}{B}$. What is $B - A$?

A local high has decided to engage students in a day of political discussion as the election season approaches. After the day has finished, all 2803 of the students at the school are asked to vote for one candidate, and the political lean of the students by grade level is documented in the table below.

	Republican (R)	Democrat (D)	Third Party (T)	Total
Freshman (F)	370	297	58	
Sophomore (So)	316	309	77	
Junior (J)	271	334	90	
Senior (Se)	232	368	81	
Total				

A student is randomly selected from those represented by the table above. Find each probability below rounding each part to the thousandths place.

A) $P(R \cup Se)$

B) $P(D \cap F)$

C) $P(So' \mid T)$

D) $P(\,(\,(F \cup J) \cap D) \mid So')$

What is $A + B + C + D$ rounded to the thousandths place?

In terms of the lower bound, A, and the upper bound, B, of a continuous uniform distribution, X, on the closed interval $[A, B]$, we have that $E(X^2) = \frac{1}{3}(A^2 + AB + B^2)$. A simplified expression for the variance of the continuous uniform distribution can be written as $\sigma^2 = \frac{(B-A)^2}{k}$ where k is a constant for all values of A and B. With the formula for the variance in mind, what is the variance of the given continuous uniform distribution bounded by the interval $[226, 237]$?

January Regional — Statistics Team: Question #7

I'd like to test how certain factors in types of fertilizer effect plant growth. I choose 3 different fertilizer brands to use in my experiment: X, Y, and Z. Within each fertilizer brand, I choose 7 different levels of nitrate content: 4 ppm, 8 ppm, 12 ppm, 16 ppm, 20 ppm, 24 ppm, and 28 ppm. Since soil type and other environmental conditions vary greatly throughout various regions of the country and state by state, I utilize plots of land in 6 major agricultural states around the country for my experiment: California, Florida, Texas, Iowa, Indiana, and Georgia. Answer each of the following.

A) How many experimental factors are there in this experiment?

B) How many blocks are there in this experiment?

C) How many total treatments are there in this experiment?

D) Which of the following is neither necessary nor possible in this experiment? Let D = 1, 2, 3, or 4 accordingly.
1. Confounding
2. Replication
3. Blinding
4. Control

What is the sum of the sample mean and the sample standard deviation of your set of answer from A, B, C, and D above rounded to the nearest hundredth?

January Regional — Statistics Team: Question #8

The Poisson Distribution is a discrete probability distribution, that tells us the probability of k (integer) discrete, independent events occurring in a certain time interval, given some expected rate, λ (not necessarily an integer), of such occurrences in that time interval. For example, if it is given that there is, on average, 2.5 major meteor strikes every 100 years and I want to find the probability that no major meteor strikes occur in the next 100 years, $\lambda = 2.5$ and $k = 0$ for the expression $P(k\ events\ in\ time\ interval) = e^{-\lambda}\frac{\lambda^k}{k!}$. For each part, round to three decimal places and if it is not appropriate to use the Poisson distribution, use -1 as that part's answer, do not use 0.

A) The probability that I get 3 pieces of mail tomorrow given that I receive, on average, 4 pieces of mail per day.

B) The probability that no major hurricanes strike Florida this hurricane season given that, on average, 0.4 hurricanes strike Florida each hurricane season.

C) The probability that 2.5 inches of rain fall today given that, on average, 2 inches of rain fall every day.

D) The probability that more than 3 major earthquakes hit the western US this year given that, on average, 1 major earthquake rocks the western US every year.

What is $A + B + C + D$ to three decimal places?

January Regional Statistics Team: Question #9

Each of the following trials involves 160,000 flips (a very large number of flips) of a fair coin. Find the probability of each of the following events rounded to two decimal places. For the following parts, you must use Normal Approximations and do not use any correction for continuity. If you do not know what a continuity correction is, do not worry about it!

A) Seeing more than 79,700 heads in 160,000 flips of a fair coin

B) Seeing between 79,700 and 80,300 heads in 160,000 flips of a fair coin

C) Seeing more than 80,300 heads in 160,000 flips of a fair coin, given that we know we will see more than 79,700.

D) After the first 70,000 flips of the coin, we found that exactly 35,500 turned up heads. What is the probability that we will see between 80,400 and 80,700 heads in the 160,000 total flips of our fair coin?

What is $A + B + C + D$ rounded to two decimal places?

January Regional Statistics Team: Question #10

When registering students for the high school's math competitions, the lead coordinator, Mr. Lapmar, misplaces the attendance list and doesn't know which division each competitor is in. However, he knows that there are exactly 9 students in each FAMAT division at his school, and he will always register the correct total number of students for each competition, which is underlined for each of the four regional competitions listed below and denoted by A, B, C, and D. Find the probability that Mr. Lapmar successfully registers the indicated total number of students for each regional competition with the indicated number of Mu's, Alpha's, and Theta's by random selection of students from the total pool of FAMAT competition students at the school. Round each part below to three decimal places.

A) Strictly 12 Total Students Registered with: 4 Mu's, 4 Alpha's, and 4 Theta's

B) Strictly 20 Total Students Registered with: 8 Mu's, 4 Alpha's, and 8 Theta's

C) Strictly 16 Students Total Students Registered with: either 4 or 8 Mu's, exactly 4 Alpha's, and the rest Theta's

D) Strictly 16 Students Total Students Registered with: 4-8 Mu's (inclusive), exactly 4 Alpha's, and the rest Theta's

What is $A + B + C + D$ rounded to three decimal places?

Prior to writing a team round question, a struggling test writer can't decide whether he wants the summary answer to be $X = -A + 2B + 2C$ or $Y = A - 2B - 2C$. The test writer does know that each of the three parts will be either 0 or 1. The table below shows all of the possible values for A, B, and C.

A	**B**	**C**	$\boldsymbol{X = -A + 2B + 2C}$	$\boldsymbol{Y = A - 2B - 2C}$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

What is the population covariance of these two distributions, X and Y, as a decimal, given that the formula for the population covariance is: $Cov(X, Y) = \sigma_{X,Y} = E[(x - E(X))(y - E(Y))]$?

Joe and Sally are playing 5-card poker. Because Joe happens to be a fan of Aces, he places a second Ace of Spaces into his original standard deck of 52 cards, shuffles the cards, and plays like normal. The second Ace of Spades is identical in every way to the original Ace of Spades. Find the probability of obtaining each of the following 5-card poker hands with this new deck of 53 playing cards.

Let $A = P(5\ of\ a\ kind = 5\ cards\ of\ the\ same\ face\ value)$

Let $B = P(Royal\ Flush = the\ following\ five\ cards\ all\ in\ one\ suit: 10, Jack, Queen, King, Ace)$

Let $C = P(Any\ given\ hand\ with\ both\ Aces\ of\ Spades)$

Let $D = P(4\ of\ a\ kind = 4\ cards\ of\ the\ same\ face\ value)$

What is $\frac{BD}{AC}$ as a simplified fraction?

At a local movie theater, only the 3 movies in the Matrix Trilogy played a particular day in appreciation of actor Keanu Reeves. Worried about attendance for those movies, the theater owner stations a worker to ask customers which singular movie they watched as they leave the theater. The population of interest was all customers who showed up that day and all customers responded to the survey. No customer watches more than 1 movie at the theater that day. The ratio of the number of viewers for each movie — "The Matrix," "The Matrix Reloaded," "The Matrix Revolutions" — is 3:2:1, respectively. The total number of moviegoers at the theater that day is 102.

Let $A =$ The Number of Distinct Letters in the Word for the Data Collection Method Used

Let $B =$ The Number of Moviegoers who Watched "The Matrix Reloaded"

Let $C =$ The Probability of Randomly Selecting a Moviegoer who Watched "The Matrix"

The determinant of the matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is 153. What is D?

Consider the following linear regression model for predicting the monthly rental cost of coastal apartments from the number of blocks to the beach.

$$\widehat{Rent} = -3520(blocks) + 10040$$

A) Residual for an apartment 1.5 blocks from the beach that costs $4600 per month.

B) Residual for an apartment 2.5 blocks from the beach that costs $1500 per month.

C) If the standard deviation of the monthly rent for these coastal apartments is $2000 per month and the standard deviation of the number of blocks these apartments are to the beach is 0.5 blocks, what is the proportion of variance explained by our above regression model? Answer as a decimal.

What is $A + B + C$?

Consider the distribution of the number of lollipops the student government sells at the school store in a given school day, which is normally distributed with a mean of A and a standard deviation of B. Also consider the distribution of the number of cupcakes that the student government sells at the store in a given school day, which is also normally distributed, but with a mean of C and a standard deviation of D. Using empirical percentages and the clues provided, find the appropriate means and standard deviations for the two given distributions.

- On 16% of school days, the student government sold fewer than 45.8 or between 58.6 and 65 lollipops.
- On 16% of school days, the student government sold fewer than 52.2 lollipops.
- On 16% of school days, the student government sold fewer than 51.9 cupcakes.
- The probability that the student government sold more than 77.5 cupcakes given that they sold more than 64.7 cupcakes in a given school day is $\frac{3}{320}$.

What is $A + B + C + D$? Do not round intermediate parts or your final answer.

January Regional — Statistics Team: Answers and Solutions

Question Number	Answer	Question Number	Answer
1	$\frac{1}{3}$	9	2.53
2	30240	10	0.326
3	16.92	11	-2.25
4	781403	12	$\frac{828}{4165}$
5	1.738	13	34
6	$\frac{121}{12}$	14	100.7744
7	16.83	15	123.3
8	-0.116		

1. 1/3

A) $E(3X+5Y) = 3E(X) + 5E(Y) = 3*10 + 5*5 = 55$

B) $E(3Y+X) = 3E(Y) + E(X) = 3*5 + 10 = 25$

C) $E(2X+2Y) = 2E(X) + 2E(Y) = 2*10 + 2*5 = 30$

D) $Var(3X-2Y) = 3^2Var(X) + 2^2Var(Y) - 2*3*2*\rho*SD(X)*SD(Y) = 289.6$

$289.6 = 9*36 + 4*4 - 2*3*2*6*2*\rho$

$-50.4 = -2*3*2*6*2*\rho$

$\rho = 0.35$

E) $Slope = \beta_1 = 0.35*\frac{SD(Y)}{SD(X)} = 0.35*\frac{2}{6} = \frac{7}{60}$

Summary: $A + \frac{E}{D} - (C+B) = 55 + \frac{\frac{7}{60}}{0.35} - (30+25) = \frac{1}{3}$

2. 30240

Crossword Solutions

2-Across: STRATIFIED, 5-Across: CONVENIENCE, 1-Down: SIMPLERANDOM, 3-Down: CLUSTER, 5-Down: SYSTEMATIC

Number of Distinct Permutations in "RILENNEET" $= \frac{9!}{3!*2!} = 30240$

3. 16.92

Where W denotes a win and L denotes a loss, to make a game fair, \$5 $*P(W) - MoneyLost*P(L) = 0$, thus making our expected profit 0. If $5*P(W) = MoneyLost*P(L)$, then $MoneyLost = \frac{5*P(W)}{P(L)} = \frac{5*P(W)}{1-P(W)}$. We shall use this expression for the parts below.

A) There are 20 multiples of three on [1,60]. $P(W) = \frac{1}{3}$ and $MoneyLost = \frac{5*\frac{1}{3}}{\frac{2}{3}} = \$2.50$

B) There are 8 multiples of seven on [1,60]. $P(W) = \frac{2}{15}$ and $MoneyLost = \frac{5*\frac{2}{15}}{\frac{13}{15}} = \$0.77$

C) There are 17 primes on [1,60]. $P(W) = \frac{17}{60}$ and $MoneyLost = \frac{5*\frac{17}{60}}{\frac{43}{60}} = \$1.98$

D) There are 42 composite integers on [1,60]. $P(W) = \frac{7}{10}$ and $MoneyLost = \frac{5*\frac{7}{10}}{\frac{3}{10}} = \$11.67$

NOTE: The integer 1 is neither prime nor composite!

Summary: $A + B + C + D = 16.92$

4. 781403

This question is intended to be done arithmetically as the numbers are too large for most calculators to easily make into a reduced fraction.

A) $binompdf(4,0.75,3) = \frac{27}{64}$

B) $geometpdf(0.75,2) + geometpdf(0.75,3) = \frac{15}{64}$

C) $0.75 * binompdf(4,0.75,2) = \frac{81}{512}$

$$P(at\ least\ 1) = 1 - P(none) = 1 - \left(1 - \frac{27}{64}\right)\left(1 - \frac{15}{64}\right)\left(1 - \frac{81}{512}\right) = 1 - \frac{37}{64} * \frac{49}{64} * \frac{431}{512} = 1 - \frac{37 * 49 * 431}{64 * 64 * 512}$$

$$= \frac{64 * 64 * 512 - 37 * 49 * 431}{64 * 64 * 512} = \frac{1315749}{2097152}$$

Since we know that we only have powers of two in the denominator and our numerator is odd, we can conclude that that the fraction is fully reduced.

Summary: $2097152 - 1315749 = 781403$

5. 1.738

Below is the table with totals filled in:

	Republican (R)	Democrat (D)	Third Party (T)	Total
Freshman (F)	370	297	58	725
Sophomore (So)	316	309	77	702
Junior (J)	271	334	90	695
Senior (Se)	232	368	81	681
Total	1189	1308	306	2803

A) $P(R \cup Se) = \frac{370+316+271+232+368+81}{2803} = 0.584$

B) $P(D \cap F) = \frac{297}{2803} = 0.106$

C) $P(So' \mid T) = \frac{58+90+81}{306} = 0.748$

D) $P(\,((F \cup J) \cap D) \mid So') = \frac{297+334}{725+695+681} = 0.300$

Summary: $A + B + C + D = 1.738$

6. $\mathbf{\frac{121}{12}}$

Because we are dealing with a continuous uniform distribution, its expected value is $\frac{A+B}{2}$.

$$Var(X) = E(X^2) - E(X)^2 = \frac{A^2 + AB + B^2}{3} - \left(\frac{A+B}{2}\right)^2 = \frac{A^2 + AB + B^2}{3} - \frac{A^2 + 2AB + B^2}{4}$$

$$= \frac{4(A^2 + AB + B^2) - 3(A^2 + 2AB + B^2)}{12} = \frac{4A^2 + 4AB + 4B^2 - 3A^2 - 6AB - 3B^2}{12} = \frac{A^2 - 2AB + B^2}{12}$$

$$Var(X) = \frac{(B-A)^2}{12}$$

For the given distribution bounded by $[226, 237]$, $Var = \frac{(237-226)^2}{12} = \frac{121}{12}$

7. 16.83

A) There are 2 experimental factors in this experiment: fertilizer brand and nitrate content. So, A = 2.

B) Since we are attempting to control for the confounding factors of soil type and other environmental factors by conducting the experiment in 6 different agricultural states, each of the 6 states is a block. So, B = 6.

C) Fertilizer brand has 3 levels: X, Y, and Z; and nitrate content had 7 levels: 4 ppm, 8 ppm, 12 ppm, 16 ppm, 20 ppm, 24 ppm, and 28 ppm; there are $3 \times 7 = 21$ total treatments. As noted above, the different states are blocks and therefore, not treatments since the experiment is essentially replicated within each state (or block). So, C = 21.

D) Some amount of confounding is always present in every experimental design, which is why we should try to control confounding factors via such methods as blocking or other forms of control. Hence, these concepts are certainly present in this experiment. Sufficient replication in the form of a sufficient number of experimental units randomly assigned to each treatment is also a requirement of any good experiment. However, since we are experimenting on plant growth based on the effect of fertilizer type, blinding is neither possible nor necessary. So, D = 3.

Summary: Entering the numbers 2, 6, 21, and 3 into a list in the calculator and running 1-Var Stats we obtain a sample mean of $\bar{x} = 8$ and a sample standard deviation of $s_x = 8.83$ and so their sum is 16.83.

8. -0.116

A) $P(k=3) = e^{-4}\frac{4^3}{3!} = 0.195$

B) $P(k=0) = e^{-0.4}\frac{0.4^0}{0!} = 0.670$

C) -1, because k must take on integer values, so the Poisson Distribution is not appropriate.

D) $P(k>3) = 1 - (e^{-1}\frac{1^0}{0!} + e^{-1}\frac{1^1}{1!} + e^{-1}\frac{1^2}{2!} + e^{-1}\frac{1^3}{3!}) = 0.019$

Summary: $A + B + C + D = -0.116$

9. 2.53

The question directions state Normal Approximations to the Binomial Distribution must be used, so calculations involving binomcdf or binompdf should not be accepted.

A) $normalcdf(79700, \infty, 160000 * 0.5, \sqrt{160000 * 0.5 * 0.5}) = 0.93$

B) $normalcdf\left(79700, 80300, 160000 * 0.5, \sqrt{160000 * 0.5 * 0.5}\right) = 0.87$

C) $P(X > 80300 \mid X > 79700) = \frac{normalcdf(80300, \infty, 160000*0.5, \sqrt{160000*0.5*0.5})}{normalcdf(79700, \infty, 160000*0.5, \sqrt{160000*0.5*0.5})} = 0.07$

D) Because we are told the first 70000 trials have been observed, we are now assessing the remaining 90000 instead of the original 160000 in our Normal Probability calculation. $80400 - 35500 = 44900$ and $80700 - 35500 = 45200$.

$P(80400 < X < 80700) = normalcdf\left(44900, 45200, 90000 * 0.5, \sqrt{90000 * 0.5 * 0.5}\right) = 0.66$

Summary: $A + B + C + D = 2.53$

10. 0.326

A) $\frac{\binom{9}{4}\binom{9}{4}\binom{9}{4}}{\binom{27}{12}} = 0.115$

B) $\frac{\binom{9}{8}\binom{9}{4}\binom{9}{8}}{\binom{27}{20}} = 0.011$

C) $\frac{\binom{9}{4}\binom{9}{4}\binom{9}{8}}{\binom{27}{16}} + \frac{\binom{9}{8}\binom{9}{4}\binom{9}{4}}{\binom{27}{16}} = 0.022$

D) $\frac{\binom{9}{4}\binom{9}{4}\binom{9}{8}}{\binom{27}{16}} + \frac{\binom{9}{5}\binom{9}{4}\binom{9}{7}}{\binom{27}{16}} + \frac{\binom{9}{6}\binom{9}{4}\binom{9}{6}}{\binom{27}{16}} + \frac{\binom{9}{7}\binom{9}{4}\binom{9}{5}}{\binom{27}{16}} + \frac{\binom{9}{8}\binom{9}{4}\binom{9}{4}}{\binom{27}{16}} = 0.178$

Summary: $A + B + C + D = 0.326$

11. -2.25

Using 2-Var Stats, $E(X) = 1.5$ and $E(Y) = -1.5$

A	**B**	**C**	$\boldsymbol{X = -A + 2B + 2C}$	$\boldsymbol{Y = A - 2B - 2C}$	$\boldsymbol{(x - E[X])(y - E[y])}$
0	0	0	0	0	-2.25
0	0	1	2	-2	-0.25
0	1	0	2	-2	-0.25
0	1	1	4	-4	-6.25
1	0	0	-1	1	-6.25
1	0	1	1	-1	-0.25
1	1	0	1	-1	-0.25
1	1	1	3	-3	-2.25

Using 1-Var Stats, $E[(x - E[X])(y - E[y])] = -2.25$

12. $\boldsymbol{\frac{828}{4165}}$

A) $P(5\ of\ a\ kind) = \frac{1}{\binom{53}{5}}$. There is only one possible 5 of a kind in our modified deck of cards.

B) $P(Royal\ Flush) = \frac{5}{\binom{53}{5}}$. There are normally 4 distinct royal flushes in a deck of 52 cards, but the second Ace of Spades grants us a second royal flush of spades.

C) $P(Any\ given\ hand\ with\ both\ Aces\ of\ Spades) = \frac{\binom{2}{2}\binom{51}{3}}{\binom{53}{5}}$. We wish to choose our two desired Aces and then 2 other arbitrary cards of the remaining 51.

D) $P(4\ of\ a\ kind) = \frac{\binom{choosing\ a\ 4\ of\ a\ kind}{of\ all\ cards\ except\ aces} + \binom{choosing\ a\ 4\ of}{a\ kind\ of\ aces}}{\binom{53}{5}} = \frac{\binom{12}{1}\binom{4}{4}\binom{49}{1} + \binom{1}{1}\binom{5}{4}\binom{48}{1}}{\binom{53}{5}}$

$$\text{Summary: } \frac{BD}{AC} = \frac{\frac{5}{\binom{53}{5}} * \frac{\binom{12}{1}\binom{4}{4}\binom{49}{1} + \binom{1}{1}\binom{5}{4}\binom{48}{1}}{\binom{53}{5}}}{\frac{1}{\binom{53}{5}} * \frac{\binom{2}{2}\binom{51}{3}}{\binom{53}{5}}} = \frac{5 * 828}{20825} = \frac{828}{4165}$$

13. 34

The table shows the number of viewers for each movie using the given ratio 3:2:1

"The Matrix"	"The Matrix Reloaded"	"The Matrix Revolutions"	Total
51	34	17	102

A) Because we are sampling the entire population, we are taking a census. There are 5 distinct letters in the word census.

B) 34, see table above.

C) $\frac{51}{102} = \frac{1}{2}$

D) $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD - CB = 153,\ 5 * D - \frac{1}{2} * 34 = 153,\ D = 34$

14. 100.7744

$Residual = y - \hat{y}$

A) $Residual = 4600 - (-3520 * 1.5 + 10040) = -160$

B) $Residual = 1500 - (-3520 * 2.5 + 10040) = 260$

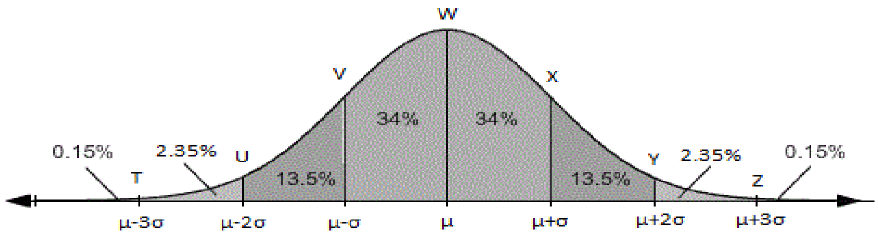
C) $Slope = r\frac{s_y}{s_x},\ -3520 = r * \frac{2000}{0.5},\ r = -0.88$

The proportion of explained variance $= r^2 = (-0.88)^2 = 0.7744$

Summary: $A + B + C = 100.7744$

15. 123.3

This is a primarily logic-based question. Chop the normal distribution into sections with labelled empirical percentages to make this question easier. The points an integer number of standard deviations from the mean of any given Normal Distribution have also been marked in the picture below for ease of explanation.



- Lollipop Distribution
  - With the first clue, we are told that 45.8 is an upper bound while the interval $[58.6, 65]$ also comprises some area of the distribution. 16% of the distribution can be seen as the interval (U, V) or $(\text{X}, \text{Y}) + (\text{T}, \text{U})$ or $(\text{Y}, \text{Z}) + (-\infty, \text{T})$ or (Z, ∞). 45.8 must either be point U or T because 45.8 is an upper bound and $P(L < 45.8) \neq 0.16$ so 45.8 cannot be V.
    - If 45.8 is U, then 58.6 must be X and 65 must be Y to capture 16% of the distribution.
      - Evaluating $Y - X$, we get $\sigma = 6.4$.
      - However, while the distance from U to X would be $3\sigma = 19.2$ according to our standard deviation, $58.6 - 45.8 \neq 12.6$ so this does not work.
    - If 45.8 is T, then 58.6 must be X and 65 must be Z to capture 16% of the distribution.
      - Evaluating $\frac{Z-X}{2}$, we get $\sigma = 3.2$.
      - Since the distance from T to X should be $4\sigma = 12.8$, and $58.6 - 45.8 = 12.8$, this works.
    - $\sigma = B = 3.2$ and $\mu = A = 55.4$
  - The second clue is not necessary to solve the problem, but we can see that 52.2 being point V reinforces our solution.
- Cupcake Distribution
  - The third clue establishes 51.9 as an upper bound for 16% of the distribution, meaning 51.9 must be V.
  - The fourth clue establishes 77.5 as a lower bound for some small percentage of the distribution as well as 64.7 as a lower bound for another percentage of the distribution. We may notice that 64.7 is the midpoint between 51.9 and 77.5. Since that is the case, 64.7 could be W and 77.5 would be X, or 64.7 could be X and 77.5 would be Z.
    - If 64.7 is W, then $\frac{P(C>77.5)}{P(C>64.7)} = \frac{.16}{.5} = \frac{8}{25} \neq \frac{3}{320}$, so this does not work.
    - If 64.7 is X, then $\frac{P(C>77.5)}{P(C>64.7)} = \frac{.0015}{.16} = \frac{3}{320}$, so this works.
    - $\sigma = D = 6.4$ and $\mu = C = 58.3$

Summary: $A + B + C + D = 55.4 + 3.2 + 58.3 + 6.4 = 123.3$