Pre-Calculus Team Solutions

1. $\frac{15\pi}{4}$ or 3.75π

2. $\frac{200}{3}$

3. -183

4. $-\frac{205}{16}$

5. $\frac{67-16\sqrt{3}}{4}$

6. $-\frac{612\pi}{5}$

7. $\frac{10\pi}{3}$

8. 18

9. -160

10. $\frac{403}{2}$

11. $\frac{3\sqrt{3}}{20}$

12. $\frac{33}{2}$ or 16.5

13. $-\frac{25\sqrt{39}}{8}\pi$

14. $\frac{25\pi}{2}$ or 12.5π

15. $20-10\sqrt{3}$

SOLUTION KEY- Part A, Part B, Part C, Part D

1. **The answer is** $\frac{\mathbf{15\pi}}{\mathbf{4}}$

PART A

Using the Rational Root Theorem, possible rational roots include $\{\pm1, \pm2, \pm3, \pm6\}$

Using synthetic division, the real roots are $\{-2, -1, 3\}$

The sum of the real roots is 0

PART B

Three digit numbers cannot begin with zero. The set of solutions is shown below:

$\{203, 230, 302, 320\}$

Of the three-digit numbers above, 3 of them are even

PART C

$C = 2\pi r = 0.25\pi$

Five revolutions is 1.25π meters

$r = \frac{distance}{time} = 1.25\pi \frac{m}{s}$

FINAL SOLUTION

$A + BC = \frac{\mathbf{15\pi}}{\mathbf{4}}$

2. **The answer is** $\frac{200}{3}$

PART A

$$s = \frac{5+6+9}{2} = 10$$

$$A = \sqrt{10(10-5)(10-6)(10-9)}$$

$$A = \sqrt{10(5)(4)(1)} = 10\sqrt{2}$$

PART B

$4x + 3 = 5 \rightarrow x = {}^{1}/_{2}$

PART D

$e_{CIRCLE} = 0$ → There are six letters.

PART C

$$2(\cos x)(\cos x - \sec x) - \sin^2 x = 2\cos^2 x - 2 - \sin^2 x$$

$$2(\cos x)(\cos x - \sec x) - \sin^2 x = 2(1 - \sin^2 x) - 2 - \sin^2 x$$

$$2(\cos x)(\cos x - \sec x) - \sin^2 x = -3\sin^2 x$$

The range of $\sin^2 x$ is $[0,1]$, hence $y_{MAX} = 0$

FINAL SOLUTION

$$\frac{A^2 - C}{BD} = \frac{200 - 0}{6\left(\frac{1}{2}\right)}$$

$$\frac{A^2}{BCD} = \frac{200}{3}$$

3. **The answer is -183**

PART A

$$M(x-2) - N(2x-1) = 5x - 7$$

$$M - 2N = 5 \qquad N - 2M = -7$$

$$Eq1 + 2 * Eq2 \rightarrow \; -3M = -9$$

$$M = 3 \rightarrow N = -1$$

$$M + N = 2$$

PART B

Using Synthetic Division

-4 |	1	6	-2	7	3
		-4	-8	40	-188
	1	2	-10	47	-185

The remainder is -185

FINAL SOLUTION

$$A + B + C = 2 - 185 + 0$$

$$A + B + C - D = -183$$

PART C

The following must be satisfied:

$$|x - 2| \le \frac{1}{4}$$

Case 1: $(x-2) \le \frac{1}{4} \rightarrow x \le \frac{9}{4}$

Case 2: $-(x-2) \le \frac{1}{4} \rightarrow x \ge \frac{7}{4}$

$x \ne 2 \rightarrow$ NO INTEGER SOLUTION

4. **The answer is $-\frac{205}{16}$**

PART A

$$\frac{2^{4041}+2^{4031}}{4^{2019}-4^{2017}} = \frac{2^{4041}+2^{4031}}{2^{4038}-2^{4034}}$$

$$\frac{2^{4041}+2^{4031}}{4^{2019}-4^{2017}} = \left(\frac{2^{4031}}{2^{4031}}\right)\frac{2^{10}+1}{2^{7}-2^{3}}$$

$$\frac{2^{4041}+2^{4031}}{4^{2019}-4^{2017}} = \frac{1024+1}{128-8}$$

$$\frac{2^{4041}+2^{4031}}{4^{2019}-4^{2017}} = \frac{205}{24}$$

FINAL SOLUTION

$$AB = \left(\frac{205}{24}\right)\left(-\frac{3}{2}\right) = -\frac{205}{16}$$

PART B

$$\sum_{n=1}^{10} \cos\left(\frac{5n}{3}\pi\right) = \cos\left(\frac{5\pi}{3}\right) + \cos\left(\frac{10\pi}{3}\right) + \cdots + \cos\left(\frac{50\pi}{3}\right)$$

$$\sum_{n=1}^{10} \cos\left(\frac{5n}{3}\pi\right) = \frac{1}{2} - \frac{1}{2} - 1 - \frac{1}{2} + \frac{1}{2} + 1 + \frac{1}{2} - \frac{1}{2} - 1 - \frac{1}{2} = -\frac{3}{2}$$

5. **The answer is** $\frac{67-16\sqrt{3}}{4}$

PART A

$$f(x) = 3\cos\left(2\left(x - \frac{5}{2}\right)\right) - 2$$

The phase shift is $^5/_2$

PART C

$$\cot(60°) = \frac{\sqrt{3}}{3}$$

$$M + N = 4$$

PART B

$$\tan(15°) = \tan\left(\frac{30°}{2}\right)$$

$$\tan\left(\frac{30°}{2}\right) = \frac{1-\cos 30°}{\sin 30°}$$

$$\tan\left(\frac{30°}{2}\right) = 2 - \sqrt{3}$$

PART D

$$2(y^2 - 4y) - 5(x^2 - 6x) = 47$$

$$2(y-2)^2 - 5(x-3)^2 = 47 + 8 - 45$$

$$\frac{(y-2)^2}{5} - \frac{(x-3)^2}{2} = 1$$

$$e = {}^c/_a = {}^{\sqrt{7}}/_{\sqrt{5}} = {}^{\sqrt{35}}/_5$$

FINAL SOLUTION

$$(AD)^2 + BC = \left(\left(\frac{5}{2}\right)\left(\frac{\sqrt{35}}{5}\right)\right)^2 + \left(8 - 4\sqrt{3}\right)$$

$$(AD)^2 + BC = \frac{35}{4} + \frac{32-16\sqrt{3}}{4} = \frac{67-16\sqrt{3}}{4}$$

6. **The answer is** $-\frac{612\pi}{5}$

PART A

$A = \frac{1}{2}BH$, where H is the altitude from a given side and B is the given side length. If true for each side, it follows the largest H occurs with the smallest side length.

From M to N: $d = \sqrt{(5-1)^2 + (-1-2)^2} = \sqrt{25}$ → *Smallest Side*

From N to O: $d = \sqrt{(-3-5)^2 + (-4+1)^2} = \sqrt{75}$

From O to M: $d = \sqrt{(1+3)^2 + (2+4)^2} = \sqrt{52}$

$$A = \pm\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 5 & -1 & 1 \\ -3 & -4 & 1 \end{vmatrix} = \pm\frac{1}{2}\left[\left(-1-(-4)\right) - 2\left(5-(-3)\right) + (-20-3)\right] = 18$$

$$A = \frac{5H}{2} = 18 \rightarrow H = {}^{36}/_5$$

PART B

$$\begin{vmatrix} -3 & 9 \\ 5 & 2 \end{vmatrix} = -6 - 45 = -51$$

PART C

$$p = \frac{\pi}{|b|} = \frac{\pi}{3}$$

FINAL SOLUTION

$$ABC = -\frac{612\pi}{5}$$

7. **The answer is** $\frac{10}{3}\pi$

PART A

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(25)\left(\frac{\pi}{3}\right) = \frac{25}{6}\pi$$

PART B

$$\begin{cases} x = 4t - 3 \\ y = 5t + 6 \end{cases} \rightarrow \begin{cases} t = \frac{x+3}{4} \\ t = \frac{y-6}{5} \end{cases}$$

$$\frac{x+3}{4} = \frac{y-6}{5}$$

$y = \frac{5x+27}{4}$ → The slope is $^5/_4$

FINAL SOLUTION

$$\frac{A}{B} = \left(\frac{25}{6}\pi\right)\left(\frac{4}{5}\right) = \frac{10}{3}\pi$$

8. **The answer is** 18

PART A

$$g(x) = 5x^2 - 13 \rightarrow x = 5\left(g^{-1}(x)\right)^2 - 13$$

$$\left(g^{-1}(x)\right)^2 = \frac{x+13}{5} \rightarrow g^{-1}(x) = \sqrt{\frac{x+13}{5}}$$

$$g^{-1}(7) = \sqrt{4} = 2$$

PART B

EQ 1: $\tan(2x) \neq \tan(-2x)$ **ODD**

EQ 2: $\cos(3x) = \cos(-3x)$ **EVEN**

EQ 3: $\sec(x)\csc(x) = 2\csc(2x)$

$2\csc(2x) \neq 2\csc(-2x)$ **ODD**

EQ 4: $|x^3| - x^2 = |(-x)^3| - (-x)^2$ **EVEN**

EQ 5: $1 - 3\sin^2(x) = \cos(2x) - \sin^2(x)$

Cosine and quadrature functions are **EVEN**

Two functions are ODD

PART C

Using transitive property

$$9 = \sqrt{(x-5)(9)}$$

$$x - 5 = 9 \rightarrow x = 14$$

FINAL SOLUTION

$$AB + C = (2)(2) + 14 = 18$$

9. **The answer is** -160

PART A

$$p = 5 - 3 = 2$$

$$y = \frac{1}{4p}(x-h)^2 + k$$

$$y = \frac{1}{8}(x-4)^2 + 6 \rightarrow y(0) = 8$$

PART B

$$x = 2 - i \rightarrow (x-2)^2 = -1$$

$$f(x) = (x^2 - 4x + 5)(x - 3)$$

$$f(x) = x^3 - 7x^2 + 17x - 15$$

$$SUM = 1 - 7 + 17 - 15 = -4$$

PART C

$$f(x) = \sin(3x)\sin(6x) = \left[\frac{1}{2}\sin(3x)\cos(3x)\right]\sin(3x)$$

$$f(x) = \frac{1}{2}\sin^2(3x)\cos(3x)$$

$$\sin(3x) \rightarrow 3x = \{0, \pi, 2\pi, \ldots\}$$

Solutions on the interval: $x = \left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$

$$\cos(3x) \rightarrow 3x = \left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\right\}$$

Solutions on the interval: $x = \left\{\frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}\right\}$

There are 5 solutions

FINAL SOLUTION

$$ABC = (8)(-4)(5) = -160$$

10. **The answer is** $\frac{403}{2}$

PART A

$$\cot\left(\frac{5\pi}{4}\right)\cos\left(\frac{2\pi}{3}\right) = (1)\left(-\frac{1}{2}\right) = -\frac{1}{2}$$

PART B

$$4(x^2 - 4x) + 3(y^2 + 2y) = -7$$

$$4(x-2)^2 + 3(y+1)^2 = 16 + 3 - 7$$

$$\frac{(x-2)^2}{3} + \frac{(y+1)^2}{4} = 1$$

$$A = \pi ab = (\sqrt{3})(2)\pi = 2\sqrt{3}\pi$$

PART C

$$3x(37 - 4x) = 234 \rightarrow x(37 - 4x) = 78$$

$$4x^2 - 37x + 78 = (4x - 13)(x - 6) = 0$$

$$x = 6$$

$$y = 37 - 4(6) = 13 \rightarrow x + y = 19$$

PART D

Turning Points: $9 - 1 = 8$

FINAL SOLUTION

$$\left(\frac{BD}{2\pi}\right)^2 - AC = \left(\frac{(2\sqrt{3}\pi)(8)}{2\pi}\right)^2 - \left(-\frac{1}{2}\right)(19)$$

$$\left(\frac{BD}{2\pi}\right)^2 - AC = (8\sqrt{3})^2 + \left(\frac{1}{2}\right)(19)$$

$$\left(\frac{BD}{2\pi}\right)^2 - AC = \frac{403}{2}$$

11. **The answer is** $\frac{3\sqrt{3}}{20}$

PART A

$$\cos C = \frac{49-(25+64)}{2(5)(8)}$$

$$\cos C = \frac{1}{2}$$

PART D

$$\csc(1860°) = \csc(60°)$$

$$\csc(60°) = \frac{2\sqrt{3}}{3}$$

PART C

$$36 + x = 9 + (x+1) + 6\sqrt{x+1}$$

$$6\sqrt{x+1} = 26$$

$$x + 1 = \frac{169}{9} \rightarrow \quad x = \frac{160}{9}$$

PART B

$$(XY)^2 = (XZ)^2 + (YZ)^2 - 2(XZ)(YZ)\cos\left(\frac{\pi}{3}\right)$$

$$49 = (XZ)^2 + 25 - 5(XZ)$$

$$(XZ)^2 - 5(XZ) - 24 = 0$$

$$XZ = 8$$

FINAL SOLUTION

$$\frac{ABD}{C} = \left(\frac{1}{2}\right)(8)\left(\frac{2\sqrt{3}}{3}\right)\left(\frac{9}{160}\right)$$

$$\frac{ABD}{C} = \frac{3\sqrt{3}}{5}$$

12. **The answer is** $\frac{33}{2}$

PART A

$$y = \frac{1}{4(3-(-2))}(x+1)^2 + 3$$

$$8 = \frac{1}{20}(x+1)^2 + 3$$

$$8 = \frac{1}{20}(m+1)^2 + 3$$

$$(m+1)^2 = 100$$

$$m = \{-11, 9\}$$

The smallest value is -11

PART B

Case 1: $x^2 - 8x - 4 < 16$

$$x^2 - 8x - 20 < 0$$

$$(x-10)(x+2) < 0$$

$$x \in (-2, 10)$$

Case 2: $x^2 - 8x - 4 > -16$

$$x^2 - 8x + 12 > 0$$

$$(x-6)(x-2) > 0$$

$$x \in (-\infty, 2)(6, \infty)$$

Six solutions: $\{-1,0,1,7,8,9\}$

The sum of the solutions is 24

PART C

$$2(x^2 - 3x) + 3(y^2 + 4y) = 45$$

$$2\left(x - \frac{3}{2}\right)^2 + 3(y+2)^2 = \frac{123}{2}$$

The center is $\left(\frac{3}{2}, -2\right)$

$$h - k = \frac{7}{2}$$

FINAL SOLUTION

$$A + B + C = -11 + \frac{7}{2} + 24 = \frac{33}{2}$$

13. **The answer is** $-\frac{25\sqrt{39}}{8}\pi$

PART A

$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ $\quad$ $\text{arccot}(-1) = \frac{3\pi}{4}$

$\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$$\arctan\left(\frac{\sqrt{3}}{3}\right) + \text{arccot}(-1) + \arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{4}$$

PART C

Use the substitution: $a = \sqrt{20 + a}$

$$a^2 - a - 20 = (a-5)(a+4) = 0$$

$a \neq -4 \rightarrow a = 5$ and the sum is 5

PART D

$$y = -1 - 2x$$

$$5x - 3(-1 - 2x) = 36$$

$$11x + 3 = 36$$

$$x = 3$$

$$y = -1 - 2(3) = -7$$

$$x + y = -4$$

PART B

$$\cos\left(\arcsin\left(\frac{5}{8}\right)\right) = \frac{\sqrt{64-25}}{8}$$

$$\cos\left(\arcsin\left(\frac{5}{8}\right)\right) = \frac{\sqrt{39}}{8}$$

FINAL SOLUTION

$$ABCD = \left(\frac{5\pi}{4}\right)\left(\frac{\sqrt{39}}{8}\right)(5)(-4)$$

$$ABCD = -\frac{25\sqrt{39}}{8}\pi$$

14. **The answer is** $\frac{25\pi}{2}$

PART A

The units digit will follow the pattern of powers of 3

$$3, 9, 7, 1, 3, \ldots$$

The pattern repeats every 4th power after the 1st power.

$$\frac{345}{4} = 86\frac{1}{4}$$

The units digit will be the 1st power after a multiple of 4. The quantity 13^{345} has a unit's digit of 3. The remainder is 3.

PART B

$$ArcCos\left(-\frac{1}{2}\right) = \frac{5\pi}{6}$$

PART C

$$|4i - 3| = \sqrt{(4)^2 + (-3)^2} = 5$$

PART D

$$\log 10 < \log 16 < \log(10^2)$$

The characteristic of a logarithm is the integral part. (*i.e.* $log\ 252 \cong 2.4014$ *has a characteristic of 2*). The characteristic of log 16 is 1.

FINAL SOLUTION

$$ABCD = (3)\left(\frac{5\pi}{6}\right)(5)(1) = \frac{25\pi}{2}$$

15. **The answer is** $20 - 10\sqrt{3}$

PART A

$$f(x) = \frac{2x^3 - 5x^2 - 8x + 20}{3x^2 + 7x + 2} = \frac{(2x^3 - 8x) - (5x^2 - 20)}{(3x+1)(x+2)}$$

$$f(x) = \frac{2x(x^2-4) - 5(x^2-4)}{(3x+1)(x+2)}$$

$$f(x) = \frac{(2x-5)(x^2-4)}{(3x+1)(x+2)} = \frac{(2x-5)(x-2)}{(3x+1)}$$

There are two asymptotes: one slant asymptote & one vertical asymptote.

PART B

$$A = \frac{1}{2}r^2\theta \text{ and } P = r\theta + 2r$$

$$r(\theta + 2) = 10 \rightarrow \theta = \frac{10}{r} - 2$$

$$r^2\theta = 12$$

$$r^2\left(\frac{2}{r} - 2\right) = 12$$

$$2r^2 - 10r + 12 = (2r - 6)(r - 2) = 0$$

The sum is five

PART C

$$4\sin(15°)\cos(75°) = 4\sin(15°)\sin\big((90 - 75)°\big)$$

$$4\sin(15°)\cos(75°) = 4\sin^2(15°)$$

$$\cos(2\theta) = 1 - 2\sin^2(\theta)$$

$$4\sin(15°)\cos(75°) = 2(1 - \cos(30))$$

$$4\sin(15°)\cos(75°) = 2 - \sqrt{3}$$

FINAL SOLUTION

$$ABC = (2)(5)\big(2 - \sqrt{3}\big)$$

$$ABC = 20 - 10\sqrt{3}$$